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Stochastic Modelling of Predator-Prey Dynamics in a Three-patch Ecosystem

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Abstract

This study presents a stochastic predator-prey model in a three-patch ecosystem, motivated by cage-based fish farming. Each patch hosts prey and predator populations, with inter-patch prey migration and unbounded variations in the population represented by stochastic terms. The model integrates logistic prey growth, predation, and mortality within a coupled system of stochastic differential equations. We assess stochastic stability using stochastic Lyapunov function methods. Numerical simulations confirm that when predator efficiency $e_i < 1$, the total population remains bounded, indicating stability. However, for $e_i > 1$, the system becomes unstable. The model also demonstrates that prey populations remain viable under low harvesting rates ($\nu_1 = \nu_2 = \nu_3 = 0.02$) and moderate noise intensities ($0.10 \leq \sigma \leq 0.90$). This work contributes to sustainable resource management by offering a robust framework for modeling predator-prey interactions in multi-patch environments.

Keywords: Stochastic differential equations (SDEs), Multi-patch ecosystem, stochastic Lyapunov function, Predator-prey model.

MSC2020 Mathematics Subject Classification: 00A71, 91B70

1 Introduction

Predator-prey interactions are fundamental processes that govern ecological dynamics, influencing population stability, community structure, and ecosystem health [5, 12, 18, 21]. Classical models, such as the Lotka–Volterra system, have provided foundational insights into these dynamics by illustrating cyclic patterns of growth and decline between predator and prey populations [18]. However, such deterministic models often oversimplify ecological reality by assuming homogeneous environments and ignoring random environmental fluctuations and spatial structure.

Effective management of ecosystems, especially fisheries, requires models that account for both spatial heterogeneity and stochasticity. Management strategies such as harvesting, if poorly planned, can lead to overexploitation and eventual population collapse [9, 11, 17]. Harvesting is typically modeled in two forms: constant yield, where a fixed amount of biomass is removed over time [4], and constant effort, where removal is proportional to the current stock and fishing effort [1]. Both strategies significantly influence the population dynamics of prey and predators and require careful calibration to avoid extinction scenarios [19, 33].

Real-world ecosystems are often spatially structured, comprising multiple patches or habitats with differing environmental conditions [15]. Species movement across these patches can affect local and global population stability. Multi-patch models, therefore, offer a more realistic framework for studying ecological interactions. Several studies have explored harvesting in predator-prey systems under deterministic frameworks [2, 3, 10, 23, 31], with harvesting treated as a dynamic variable [27, 28, 29, 30].

Incorporating stochasticity into ecological models allows for the capture of unpredictable influences such as environmental variability and demographic fluctuations [13, 26]. Stochastic models, typically formulated as stochastic differential equations (SDEs), have been applied to fisheries to evaluate sustainability under environmental noise [14, 20]. These models reveal that increased variance in population due to stochastic perturbations can destabilize otherwise stable systems. However, most studies remain limited to one or two spatial zones, which constrains their applicability to real ecosystems characterized by more complex spatial dynamics.

Cage farming has grown rapidly in Lake Victoria and other regions. Despite its growth, there is limited research addressing the ecological and mathematical modeling of predator-prey dynamics within such spatially confined systems. In particular, the role of inter-cage prey migration, environmental noise, and optimal harvesting remains underexplored [24].

While some models have considered optimal harvesting in dual-zone ecosystems, such as free fishing and protected areas [8, 25], these approaches often assume constant harvesting efforts and simplified migration dynamics. Realistically, harvesting is influenced by market, regulatory, and technological factors, and migration patterns are often complex and nonlinear.

To address these gaps, this study develops a stochastic predator-prey model within a three-patch ecosystem that reflects the spatial structure of cage aquaculture. The model incorporates logistic prey growth, predator-prey interaction, inter-patch migration, and stochastic noise. Unlike previous studies, it extends stochastic modeling to three interacting patches, thereby offering a more realistic framework for analyzing ecological dynamics and informing sustainable harvesting practices.

2 The Model

The total fish population under study, denoted by $V(t)$, comprises six interacting classes across three distinct patches: prey population in patch-1 (N_1), predator population in patch-1 (M_1), prey population in patch-2 (N_2), predator population in patch-2 (M_2), prey population in patch-3 (N_3), and predator population in patch-3 (M_3). Each patch is enclosed by mesh structures that allow constant migration of fingerlings of the prey species between patches, resulting

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in interconnected dynamics. The model is developed under the assumption that each patch within the ecosystem is homogeneous in terms of environmental conditions (such as water temperature, pH level, water flow, and exchange rates of waste products) and resource availability (food supply, water quality). The prey population in each patch grows logistically in the absence of predators, with intrinsic growth rate r_i and carrying capacity K_i, while predators feed on prey at a rate a_i, contributing to predator growth at rate γ_i. Predators also experience natural mortality at rate μ_i, and both prey and predators are subject to harvesting at controlled rates ν_i. Prey migration is governed by directional rates α_1 through α_6, with each cage (patch) supporting a different population due to spatial heterogeneity.

The fish population is further assumed to exhibit stochastic fluctuations resulting from unbounded variations in the population captured using white noise terms $d\eta_t$, $d\xi_t$, and $d\epsilon_t$, modeled as Wiener processes $\sim N(0, dt)$. This leads to a system of stochastic differential equations:

$$\begin{aligned}
dN_1 &= \left(r_1 N_1 \left(1 - \frac{N_1}{K_1}\right) - a_1 N_1 M_1 - \nu_1 N_1 + \alpha_4 N_2 + \alpha_3 N_3 - (\alpha_1 + \alpha_6) N_1\right) dt + \sigma_1 N_1 d\eta_t,\\
dM_1 &= (\gamma_1 N_1 M_1 - \mu_1 M_1 - \nu_1 M_1) dt + \sigma_1 M_1 d\eta_t,\\
dN_2 &= \left(r_2 N_2 \left(1 - \frac{N_2}{K_2}\right) - a_2 N_2 M_2 - \nu_2 N_2 + \alpha_1 N_1 + \alpha_5 N_3 - (\alpha_2 + \alpha_4) N_2\right) dt + \sigma_2 N_2 d\xi_t,\\
dM_2 &= (\gamma_2 N_2 M_2 - \mu_2 M_2 - \nu_2 M_2) dt + \sigma_2 M_2 d\xi_t,\\
dN_3 &= \left(r_3 N_3 \left(1 - \frac{N_3}{K_3}\right) - a_3 N_3 M_3 - \nu_3 N_3 + \alpha_2 N_2 + \alpha_6 N_1 - (\alpha_3 + \alpha_5) N_3\right) dt + \sigma_3 N_3 d\epsilon_t,\\
dM_3 &= (\gamma_3 N_3 M_3 - \mu_3 M_3 - \nu_3 M_3) dt + \sigma_3 M_3 d\epsilon_t. \qquad (1)
\end{aligned}$$

with initial conditions:

$$\begin{aligned}
N_1(0) &= N_1 \geq 0,\\
N_2(0) &= N_2 \geq 0,\\
N_3(0) &= N_3 \geq 0,\\
M_1(0) &= M_1 \geq 0,\\
M_2(0) &= M_2 \geq 0,\\
M_3(0) &= M_3 \geq 0. \qquad (2)
\end{aligned}$$

The total population at time t is given by:

$$V(t) = N_1(t) + N_2(t) + N_3(t) + M_1(t) + M_2(t) + M_3(t) \qquad (3)$$

3 Model Analysis

3.1 Positivity of the model

The variables $N_1, M_1, N_2, M_2, N_3, M_3$ represent populations, so they should remain non-negative.

Lemma 1. *Under initial conditions (2), all the solutions* $N_1, M_1, N_2, M_2, N_3, M_3$ *of the system (1) remain nonnegative for* $t \geq 0$

Proof. For the prey population, the equations for N_1, N_2, N_3 are of the form;

$$\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i}{K_i}\right) - (\text{non-negative terms}) + (\text{migration terms}).$$

The term $r_i N_i \left(1 - \frac{N_i}{K_i}\right)$ is non-negative when $N_i \geq 0$, as the logistic growth ensures $N_i \geq 0$. Predation $(-a_i N_i M_i)$ and harvesting $(-\nu_i N_i)$, act as sinks but cannot make N_i negative if $N_i \geq 0$. Migration input terms $(\alpha_{ij} N_j)$ are non-negative, adding to the population. Thus, if $N_i(0) \geq 0$, then $\frac{dN_i}{dt}$ ensures $N_i(t) \geq 0$ for all $t \geq 0$.

For the predator population, the equations for M_1, M_2, M_3 are of the form;

$$\frac{dM_i}{dt} = \gamma_i N_i M_i - (\mu_i + \nu_i) M_i.$$

The term $\gamma_i N_i M_i$ is non-negative for $N_i \geq 0$ and $M_i \geq 0$. The mortality terms $(-\mu_i M_i$ and $-\nu_i M_i)$ act as sinks but cannot make M_i negative if $M_i \geq 0$. Thus, if $M_i(0) \geq 0$, then $\frac{dM_i}{dt}$ ensures $M_i(t) \geq 0$ for all $t \geq 0$.

The initial conditions $N_i(0) \geq 0$ and $M_i(0) \geq 0$ hold, the structure of the equations ensures that all solutions $N_i(t)$ and $M_i(t)$ remain non-negative for all $t \geq 0$. Thus, positivity of the system. ☐

3.2 Boundedness of the model

Proposition 1. *Under initial conditions (2), the total population function*

$$V(t) = \sum_{i=1}^{3} (N_i(t) + M_i(t))$$

remains bounded for all $t \geq 0$.

Proof. To prove the boundedness of the solutions for the given system of differential equations, we used the LaSalle Invariance Principle [16] and analyzed the structure of the system. We need to sought a function that is non-negative and has the following properties; decreases along the trajectories of the system, and bounds the populations, meaning it does not increase without bound as $t \to \infty$.

Let the total population of the system be represented by the function;

$$V(t) = N_1(t) + N_2(t) + N_3(t) + M_1(t) + M_2(t) + M_3(t)$$

where $N_i(t)$ and $M_i(t)$ denote the prey and predator populations in patch i respectively, for $i = 1, 2, 3$. We aim to show that $V(t)$ remains bounded for all $t \geq 0$. This function, $V(t)$, was always non-negative because populations cannot be negative.

Differentiating V(t) with respect to time gives;

$$\frac{dV}{dt} = \frac{dN_1}{dt} + \frac{dM_1}{dt} + \frac{dN_2}{dt} + \frac{dM_2}{dt} + \frac{dN_3}{dt} + \frac{dM_3}{dt}$$

Substituting the system of equations into this derivative;

$$\begin{aligned}\frac{dV}{dt} = {} & \left(r_1 N_1 \left(1 - \frac{N_1}{K_1}\right) - a_1 N_1 M_1 - \nu_1 N_1 + \alpha_4 N_2 + \alpha_3 N_3 - (\alpha_1 + \alpha_6) N_1\right) \\ & + (\gamma_1 N_1 M_1 - \mu_1 M_1 - \nu_1 M_1) \\ & + \left(r_2 N_2 \left(1 - \frac{N_2}{K_2}\right) - a_2 N_2 M_2 - \nu_2 N_2 + \alpha_1 N_1 + \alpha_5 N_3 - (\alpha_2 + \alpha_4) N_2\right) \\ & + (\gamma_2 N_2 M_2 - \mu_2 M_2 - \nu_2 M_2) \\ & + \left(r_3 N_3 \left(1 - \frac{N_3}{K_3}\right) - a_3 N_3 M_3 - \nu_3 N_3 + \alpha_2 N_2 + \alpha_6 N_1 - (\alpha_3 + \alpha_5) N_3\right) \\ & + (\gamma_3 N_3 M_3 - \mu_3 M_3 - \nu_3 M_3)\end{aligned}$$

The right-hand side of the equation is a sum of terms that include logistic growth terms which are self-limiting and ensure prey growth does not exceed the carrying capacity K_i, negative interaction terms like $-a_i N_i M_i$, representing predation and reduce prey population when predator numbers increase, mortality and harvesting terms which continuously act to reduce the population sizes, and migration terms which redistribute prey among patches without introducing unbounded growth. The logistic terms inherently bound the prey population by their respective carrying capacities. The mortality and harvesting terms act as sinks, further limiting unbounded growth. Predation introduces nonlinear decay effects, ensuring that even in the presence of high prey numbers, predator pressure helps regulate population sizes. The predator equations similarly contain death and harvesting terms that prevent unbounded growth.

Formalizing this by constructing a Lyapunov-like function,

$$V(t) = \sum_{i=1}^{3} \left(N_i(t) + M_i(t)\right),$$

and analyze its derivative,

$$\frac{dV}{dt} \leq C - DV(t),$$

where $C > 0$ is a constant representing the maximum cumulative contribution from logistic and migration terms, and $D > 0$ is a constant representing the decay effect due to mortality, harvesting, and predation. This inequality implies that $V(t)$ grows at a rate bounded above by a linear function that decreases in V, leading to the conclusion (by the Comparison Theorem [6]) that;

$$V(t) \leq \max\left(V(0), \frac{C}{D}\right), \quad \text{for all } t \geq 0.$$

Hence, all individual population components $N_i(t)$ and $M_i(t)$ are bounded above by finite constants. □

3.3 Stability Analysis

A standard Stochastic Differential Equations (SDEs) takes the form;

$$dx_i = f_i(x)dt + g_i(x)dW_i(t), \quad i = 1, 2, 3, ..., n$$

where $f_i(x)$ captures deterministic dynamics, $g_i(x)$ incorporates stochastic effects, and $W_i(t)$ are Wiener processes representing the random fluctuations.

We can rewrite (1) in a general form as;

$$dN_i = f_i(N, M)\, dt + g_i(N, M)\, dW_i(t), \quad i = 1, 2, 3$$

$$dM_i = h_i(N, M)\, dt + k_i(N, M)\, dW_i(t), \quad i = 1, 2, 3$$

where $f_i(N, M)$ is the deterministic component of N_i, $g_i(N, M)$ is the stochastic component of N_i, $h_i(N, M)$ is the deterministic component of M_i, $k_i(N, M)$ is the stochastic component of M_i, and $dW_i(t)$ represent the Wiener processes.

We define the Lyapunov function as;

$$V(N, M) = \sum_{i=1}^{3} \left(\frac{N_i^2}{2} + \frac{M_i^2}{2} \right).$$

which measures the "energy" or total population in the system.

Computing the partial derivatives of $V(N, M)$, we have;

$$\begin{aligned} \frac{\partial V}{\partial N_i} &= N_i, \\ \frac{\partial V}{\partial M_i} &= M_i, \\ \frac{\partial^2 V}{\partial N_i^2} &= 1, \\ \frac{\partial^2 V}{\partial M_i^2} &= 1. \end{aligned}$$

The first 2 equations (this applies to equations for dN_2, dM_2, dN_3, dM_3) of SDEs for equation (1) can be re-written as;

$$dN_1 = f_1(N, M)\, dt + g_1(N, M)\, dW_1,$$

$$dM_1 = h_1(N, M)\, dt + k_1(N, M)\, dW_1,$$

where;

$$\begin{aligned} f_1(N, M) &= r_1 N_1 \left(1 - \frac{N_1}{K_1} \right) - a_1 N_1 M_1 - \nu_1 N_1 + \alpha_4 N_2 - (\alpha_1 + \alpha_6) N_1, \\ g_1(N, M) &= \sigma_1 N_1, \\ h_1(N, M) &= \gamma_1 N_1 M_1 - \mu_1 M_1 - \nu_1 M_1, \\ k_1(N, M) &= \sigma_1 M_1. \end{aligned}$$

Applying the Itô's Lemma [22], the stochastic differential of $V(N, M)$ becomes;

$$dV = \sum_{i=1}^{3} \left(\frac{\partial V}{\partial N_i} \, dN_i + \frac{\partial V}{\partial M_i} \, dM_i \right) + \frac{1}{2} \sum_{i=1}^{3} \left(\frac{\partial^2 V}{\partial N_i^2} (dN_i)^2 + \frac{\partial^2 V}{\partial M_i^2} (dM_i)^2 \right).$$

Substituting $dN_i = f_i \, dt + g_i \, dW_i$ and $dM_i = h_i \, dt + k_i \, dW_i$, we have;

$$dV = \sum_{i=1}^{3} (N_i f_i + M_i h_i) \; dt + \sum_{i=1}^{3} (N_i g_i + M_i k_i) \; dW_i + \frac{1}{2} \sum_{i=1}^{3} \left(g_i^2 + k_i^2 \right) \, dt.$$

where, $N_i f_i$ and $M_i h_i$ contribute to the deterministic drift term, $N_i g_i$ and $M_i k_i$ are associated with the stochastic terms, $g_i^2 + k_i^2$ contribute to the second-order term.

Computing each term, we have that;
For $i = 1$(similar for $i = 2$ and $i = 3$;

$$\begin{aligned}
f_1(N, M) &= r_1 N_1 \left(1 - \frac{N_1}{K_1} \right) - a_1 N_1 M_1 - \nu_1 N_1 + \alpha_4 N_2 - (\alpha_1 + \alpha_6) N_1, \\
g_1(N, M) &= \sigma_1 N_1, \\
h_1(N, M) &= \gamma_1 N_1 M_1 - \mu_1 M_1 - \nu_1 M_1, \\
k_1(N, M) &= \sigma_1 M_1.
\end{aligned}$$

The deterministic term in dV becomes;

$$N_1 f_1 + M_1 h_1 = N_1 \left[r_1 N_1 \left(1 - \frac{N_1}{K_1} \right) - a_1 N_1 M_1 - \nu_1 N_1 + \alpha_4 N_2 - (\alpha_1 + \alpha_6) N_1 \right] + M_1 \left[\gamma_1 N_1 M_1 - \mu_1 M_1 - \nu_1 M_1 \right].$$

The stochastic term in dV becomes;

$$N_1 g_1 + M_1 k_1 = N_1 (\sigma_1 N_1) + M_1 (\sigma_1 M_1).$$

The second-order term in dV becomes;

$$g_1^2 + k_1^2 = (\sigma_1 N_1)^2 + (\sigma_1 M_1)^2.$$

Therefore, the full equation for $i = 1$ is;

$$\begin{aligned}
dV_1 = & \left(r_1 N_1^2 - \frac{r_1 N_1^3}{K_1} - a_1 N_1^2 M_1 - \nu_1 N_1^2 + \alpha_4 N_1 N_2 - (\alpha_1 + \alpha_6) N_1^2 + \gamma_1 N_1 M_1^2 - \mu_1 M_1^2 - \nu_1 M_1^2 \right) \\
& dt + \left(\sigma_1 N_1^2 + \sigma_1 M_1^2 \right) dW_1 + \frac{1}{2} \left(\sigma_1^2 N_1^2 + \sigma_1^2 M_1^2 \right) dt
\end{aligned}$$

For $i = 2$ and $i = 3$, with corresponding terms for f_2, h_2, g_2, k_2 and f_3, h_3, g_3, k_3, the general form can be represented as;

$$dV = \sum_{i=1}^{3} \left[(N_i f_i + M_i h_i) \, dt + (N_i g_i + M_i k_i) \, dW_i + \frac{1}{2} \left(g_i^2 + k_i^2 \right) dt \right]$$

To ensure stochastic stability, the expected change in the Lyapunov function, $V(N, M)$, should be negative;

$$\mathbb{E} \left[\frac{dV}{dt} \right] = \sum_{i=1}^{3} \left[N_i f_i + M_i h_i + \frac{1}{2} (g_i^2 + k_i^2) \right] < 0.$$

This is achieved when; drift terms dominate and are negative, harvesting rates ν_i and mortality rates μ_i are large enough, noise intensities σ_i are not too large, and predator efficiency γ_i and prey growth r_i are not excessively high.

Numerical results

Following the analytic derivation of the Lyapunov function and the conditions for stochastic stability, we now present numerical simulations to illustrate the system's behavior under different parameter regimes.

The system was numerically simulated and the Lyapunov function (combined "energy" of all prey and predator populations) computed as;

$$V(t) = \sum_{i=1}^{3} \frac{N_i(t)^2 + M_i(t)^2}{2}$$

To ensure stochastic stability, we need to show that $V(t)$ remains bounded (decreases in expectation). Lyapunov Function,

$$LV < 0 \Rightarrow (e_i - 1) < 0 \Rightarrow e_i < 1$$

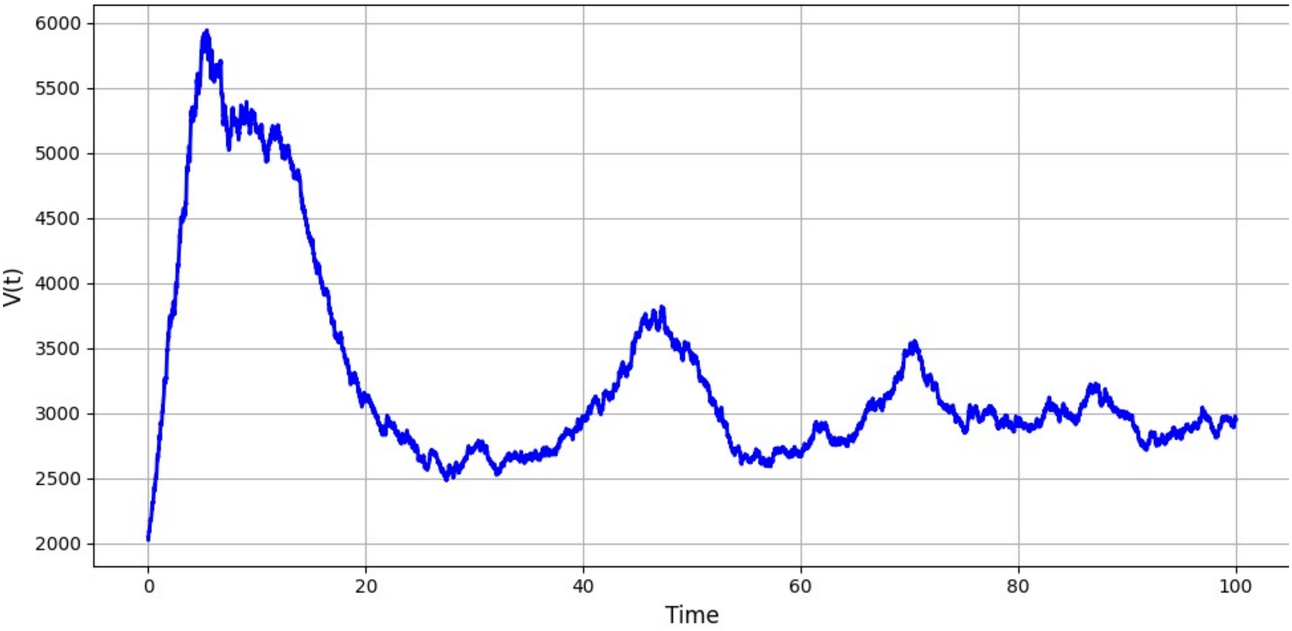

Figure 1: Lyapunov Function V(t) over Time when $e_i < 1$.

In Figure 1, when $e_i < 1$, $V(t)$ stays bounded indicating stochastic stability. This implies that the model is resilient to environmental noise and the populations tend to stabilize despite randomness.

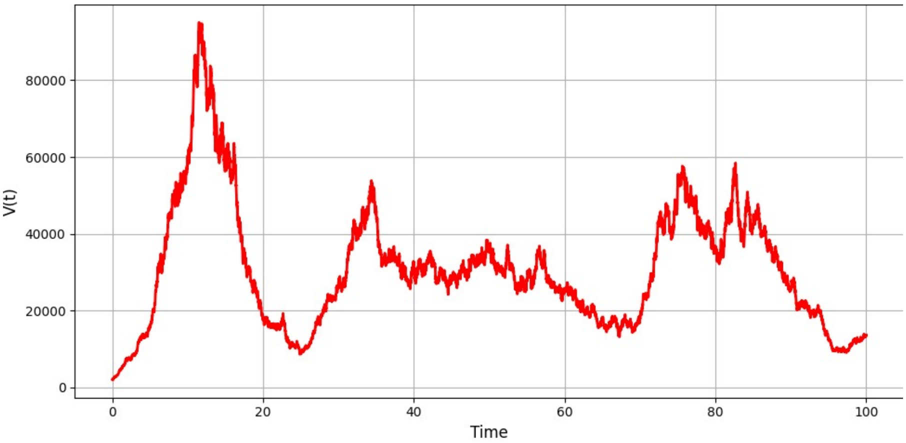

Figure 2: Lyapunov Function V(t) over Time when $e_i > 1$.

In Figure 2, when $e_i > 1$, $V(t)$ grows without bound indicating that the system is unstable. This implies that low harvesting rates or high noise intensity destabilize the system emphasizing the importance of harvesting and mortality mechanisms in maintaining ecological equilibrium, especially under environmental uncertainty.

3.4 Numerical Simulation

To illustrate the dynamic behavior of the model under stochastic influences, we performed numerical simulations using Python. The parameter values used are summarized in the table below:

Table 1: Parameter values

Parameter/Variable	**Description**	**Value**	**Source**
ν_1, ν_2, ν_3	Harvesting rates in patches 1–3	0.02	Estimated
K_1, K_2, K_3	Carrying capacities in patches 1–3	10, 8, 6	Estimated
a_1, a_2, a_3	Predation rates in patches 1–3	0.2, 0.3, 0.4	Estimated
r_1, r_2, r_3	Prey growth rates in patches 1–3	0.5, 0.4, 0.3	[7]
μ_1, μ_2, μ_3	Predator death rates in patches 1–3	0.1	[32]
$\gamma_1, \gamma_2, \gamma_3$	Predator growth rates in patches 1–3	0.01	Estimated
σ_i	Intensity of stochastic fluctuations, $i = 1, 2, 3$	Variable	Estimated
α_1 to α_6	Prey migration rates between patches	0.03	Estimated

Stochastic analysis was carried out to understand how unbounded variations in the population influence fish population dynamics in the three-patch ecosystem. Simulations were performed with varying noise intensities, and the resulting population trajectories for $N_1, M_1, N_2, M_2, N_3, M_3$ are displayed below.

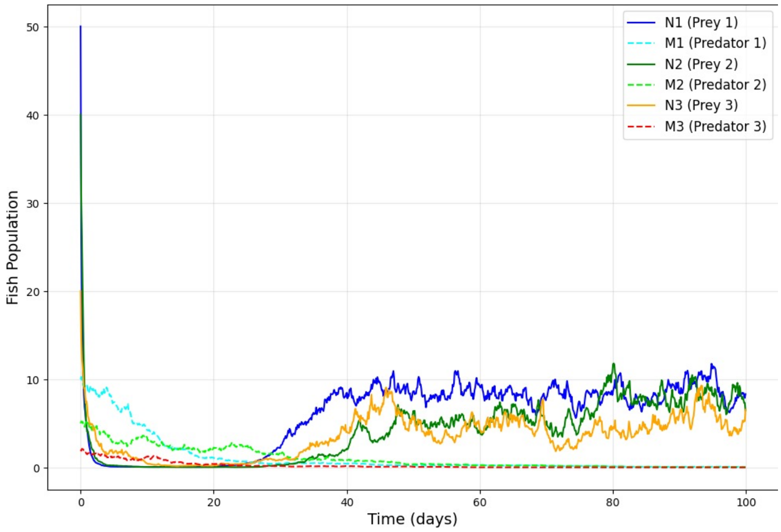

Figure 3: Stochastic dynamics with noise intensities 0.1, 0.05, 0.15, 0.1, 0.2, 0.15 for the six fish populations.

Figure 3 shows minimal oscillatory behavior across all populations, suggesting that the system experiences low influence at the given noise intensities.

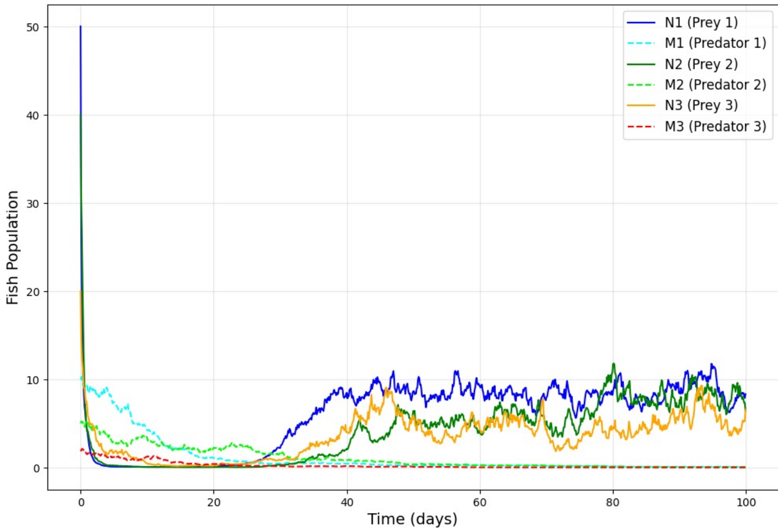

Figure 4: Stochastic dynamics with noise intensities 0.15, 0.1, 0.2, 0.15, 0.25, 0.2 for the six fish populations.

In Figure 4, the fish populations exhibit moderate oscillations, indicating a greater impact of stochasticity on system dynamics.

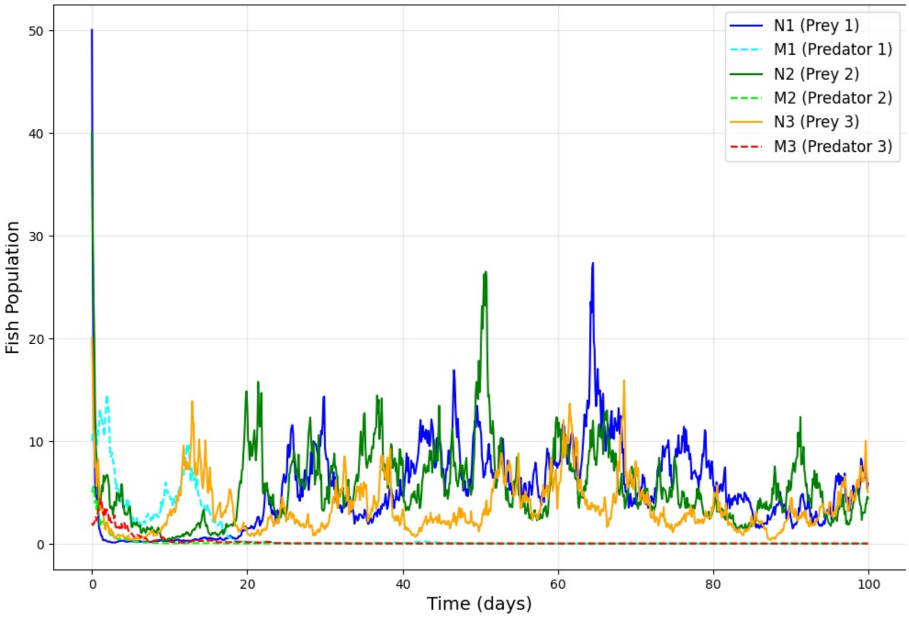

Figure 5: Stochastic dynamics with noise intensities 0.45, 0.4, 0.5, 0.45, 0.55, 0.5 for the six fish populations.

Figure 5 reveals more pronounced oscillations in all fish populations, reflecting increased sensitivity to environmental noise.

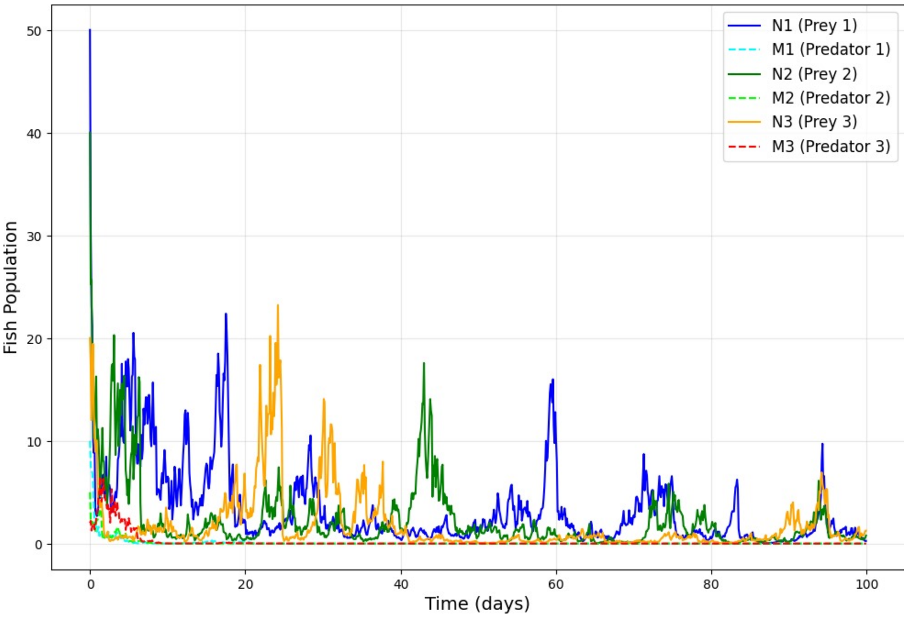

Figure 6: Stochastic dynamics with noise intensities 0.75, 0.8, 0.85, 0.75, 0.9, 0.85 for the six fish populations.

In Figure 6, strong and highly oscillatory patterns are evident, indicating high stochastic influence and dynamic instability in the ecosystem under such intense environmental variation.

4 Discussion

This study set out to develop and analyze a stochastic predator-prey model within a three-patch aquatic ecosystem using systems of stochastic differential equations (SDEs). The model incorporated ecological assumptions including logistic prey growth, distinct population dynamics per cage, and constant migration of prey fingerlings between patches. Migration rates were defined to reflect directional movement between patches, while unbounded variations in the population was captured through white noise terms acting on both prey and predator populations. Using the stochastic Lyapunov function method, the analysis revealed that system stability is significantly influenced by predator efficiency. When the efficiency parameter $e_i < 1$, the total population $V(t)$ remains bounded over time, indicating stochastic stability. Conversely, when $e_i > 1$, the population exhibits unbounded growth, signaling system instability.

Numerical simulations conducted in Python supported the analytical findings and demonstrated how varying noise intensities, harvesting rates, and migration patterns affect system behavior. At low harvesting rates ($\nu_1 = \nu_2 = \nu_3 = 0.02$) and moderate noise intensities (σ between 0.10 and 0.90), prey populations remained viable and relatively stable. However, higher noise levels or excessive harvesting led to increased oscillations and a greater risk of population decline or extinction. These findings highlight the importance of carefully managing both environmental variability and resource extraction practices. The model provides useful insights for designing sustainable harvesting strategies and improving ecological resilience in spatially structured aquatic systems.

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AUTHORS' CONTRIBUTIONS

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

Conflict of Interest

Authors have declared that no competing interests exist.

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