

**MODELLING QUEUING PHENOMENON OF HOSPITAL CONGESTION WITH
APPLICATION TO MOI TEACHING AND REFERRAL HOSPITAL**

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Research Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in Statistics of Masinde Muliro University of Science and Technology.

August, 2023

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ABSTRACT

Queuing theory is the mathematical study of waiting lines or queues, a phenomenon which is very common in any service provision facility such as supermarkets, banks, hospitals, library, transport and telecommunication lines among others. Queues occur when requests for service is greater than service providers. Long queues are undesirable in any service provision point because they result in time wastage, anxiety and boredom leading to poor customer satisfaction, poor sales and reduced profits, and most disastrously death if a critical patient is not attended to in time. The research models queuing scenario at Moi Teaching and Referral Hospital, one of the major health referral facilities in Kenya, located in Eldoret Town of Uasin Gishu County. The Government of Kenya introduced agenda four flagship development programs in 2017 one of them being provision of universal health care. This is informed by the rapidly growing population in Kenya coupled with spread of diseases, high birth rate and low life expectancy rates. The above led to overcrowding in Hospitals' Emergency Departments thus threatening the achievement of the health agenda and all the agendas in general. The emergence of Covid-19 posed great health challenge worldwide whereby health care facilities were fully stretched with no space to admit new critical patients. This motivated us to model the process as a queuing system with heterogeneous server pools, where the pools represent the wards and servers are beds. We analyzed this system under various queue-architectures and routing policies, in search for fairness and optimum operational performance so as to enhance the level of access to health care in the facility. Focusing only on the stream of emergency patients, a queuing network model interaction between the Emergency Department and Internal Wards, which was believed to cause a major proportion of the blocking at the Emergency Department. Through the use of secondary data from the Hospital and existing models such as Kendall's, Erlang's, Little's Law and de Bruin, various ward/unit operating characteristics and sufficient bed count were determined so as to guarantee certain access standards to care. Results have shown that redistribution of beds among the wards is significant in reducing congestion in the facility.

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ABBREVIATIONS AND SYMBOLS

A(t)	Distribution function of the inter-arrival time
A/B/m/K/n/D	Kendall's notation
ACF	Acute Care Facility
ADA-R	Alcohol Drug Abuse Rehabilitation
ALC	Alternative Level of Care
ALOS	Average Length of Stay
B(t)	Distribution function of the service time
CAR-E	Centre for Assault Recovery - Eldoret
COVID-19	Corona Virus Disease - 2019
CSSD	Central Supply Sterilization Department
CCU	Cardiac Care Unit
D	Service discipline
ECF	Extended Care Facility
ED	Emergency Department
FCFS	First Come First Serve
HDU	High Dependency Unit
ICU	Intensive Care Unit
i.i.d	Independently and Identically Distributed
IW	Internal Ward
K	System capacity/buffers
m	Number of servers
M	Exponential distributed random variables/Markovian
M/M/c/c	Erlang Loss model
M/G/m	m-server system with Poisson arrival
M/M/1	Single server queue
M/M/s	Multi server queue
MU	Medical Unit
MTRH	Moi Teaching and Referral Hospital
n	Population size, number of sources of customers/patients
P	Probability of an event occurring
PW	Private Wing
SPT	Shortest processing time
TAR	Target Access Rate
TOR	Target Occupancy Rate
TTA	Target Time to Access
T	Random variable representing inter-arrival time
UHC	Universal Health Care
X	Random variable representing service time

LIST OF MATHEMATICAL SYMBOLS

λ	the arrival rate
K	the number of server pools
μ_i	the service rate of a server in pool i
N_i	the number of servers in pool i
N	the total number of servers in the system: $N = \sum_{i=1}^K N_i$
E	expectation
P	probability
P_0	probability that the system is idle
P_n	probability that there are n items in the system
P_w	Probability that arriving unit has to wait for service
W_q	the stationary waiting time in the centralized queue
L_q	the stationary centralized queue-length
L	total number of items in the system (Queue Length)
ρ	the total traffic intensity
W	waiting time plus service time (time spent in the system)

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CHAPTER 1

INTRODUCTION

1.1 Background of the study

In the last two decades the service sector has grown significantly and accounts for approximately 50% of the Kenyan economy, and similarly in many other African countries according to World Bank. The service sector covers a wide spectrum of activities, e.g. professional, financial and government services. The Government of Kenya introduced Big Four Development Agenda in 2017 namely food security, affordable housing, manufacturing and job creation and most importantly provision of universal health care to its ever surging population which stood at 52.57 million (2019 Census). The research focuses on a very important part of the service sector - the health care system, and in particular on hospitals. A hospital is an institution for health care, which is able to provide long term patient stays. Over the years, hospitals have been successful in using medical and technical innovations to deliver more effective clinical treatments, while reducing patients' time spent in the hospital. However, hospitals are typically rife with inefficiencies and delays, thus present a propitious ground for many research projects in numerous science fields, and in the Operations Research field in particular.

Hospitals include numerous medical units specializing in different areas of medicine, for example, internal, surgery, intensive care, obstetrics, and so forth. In most large hospitals, there are several similar medical units operating in parallel. The research focuses on the ED and its interface with twenty eight IWs of MTRH. The ED caters immediate threats to health and provides emergency medical services. Thus the proper functioning of the ED is of utter importance, and its overcrowding can cause an inability to admit new patients and ambulances diversions [7] for consequences of ED congestion.

A patient arriving to the ED undergoes registration, diagnostic testing, and basic treatment and then is either dismissed or admitted to stay, the latter if doctors decide on hospitalization, in which case the patient is transferred to the appropriate medical unit. We focus on admitted internal patients, specifically on the process from the decision of hospitalization till admission to the IW. Two main problems could arise in the process: patients' waiting times in the ED for a transfer to the IWs could be long, and patients' allocation to the wards need not be fair.

The main goal of this research is to model and analyse the ED-IW process. Practically, to study the process of patient flow in MTRH and the associated challenges in details. Theoretically, to model the ED-to-IW process as a queuing system with heterogeneous server pools: the pools represent the wards and servers are beds. The system is analysed under various operating measures and routing policies, in search for fairness and good operational performance.

1.2 Statement of the problem

A hospital is an institution for health care, which is able to provide long term patient stays. Sundarapandian [12] states that hospitals are increasingly aware of the need to use their resources as efficiently as possible in order to continue to assure their institutions' survival and prosperity. MTRH serves residents of Western Kenya region with a population of approximately 24 Million, therefore long queues and congestion at the ED is inevitable due to the strained resources like beds and medical staff. Referral hospitals are typically rife with inefficiencies and delays, present a propitious ground for numerous research projects in Science and Operations Research fields on how such operational bottlenecks can be solved. Previous studies in queue analysis in a hospital set up did not adequately address the aspect of blocking associated with insufficiency of facilities such as beds in key departments. It is against this backdrop that the study intends to analyze and come up with a better model that eases congestion at the facility. This will be achieved by having an adequate model

of patient flows to and between the different departments of the Hospital.

1.3 Main Objective

The main objective of the study is to model the ED-IW process at MTRH, in order to minimize congestion.

1.4 Specific Objectives

The specific objectives of this study are as follows;

- (i) To analyze patients flow process between ED and IW of MTRH.
- (ii) To model bed requirements per ward/unit.
- (iii) To fit the data in the model in order to estimate bed requirement per ward/unit.

1.5 Significance of the Study

Armony M. *etal* [2] noted that accurate estimation and forecasting of parameters were prerequisites for consistent service levels and efficient operation and that though a lot had been done in statistical inference and forecasting, comparatively little had been devoted to queuing processes, particularly queuing for services in health care settings. The study shall be significant in the following ways: First it will guide hospital management in formulating policies that will result in enhanced patient service in the ED. Second, it serves as a basis of further research on use of simulation modeling in other sectors of the economy. Third, it will add to the already existing knowledge on the use of simulation modeling in health care settings.

CHAPTER 2

THEORETICAL BACKGROUND AND LITERATURE REVIEW

2.1 Introduction

In this chapter, a review of literature on queuing models related to this study is done. In the past, queuing theory has been effectively used in such areas of health care modeling as staff scheduling, policy making for example determining how prioritizing certain groups of patients affect waiting times, and bed requirements.

2.2 Waiting Lines (Queues)

Queuing can be defined as waiting to be served. Waiting lines form because clients or requests arrive at the servicing function, or server, faster than they can be served. Waiting lines result because clients do not arrive at a constant rate, nor are they served in. Decisions and management of waiting lines are based on average customers arrivals and service times.

Waiting to be attended to is undesirable. Models have been developed to help managers understand and make better decisions concerning the operation of waiting lines. Efficiency and effectiveness of outpatient services have many dimensions, but an important aspect is excessive waiting time, which is a major complaint of patients [3]. Waiting time in outpatient clinics has been documented to be a source of dissatisfaction among patients [1, 11, 13, 24] argues that this is the one consistent feature of dissatisfaction that has been expressed without patient service. The realization that patient waiting time is directly related to service quality prompted large number of studies to focus on how to reduce this time [29]. This and other studies used Accident and Emergency simulation models to evaluate the impact operational changes such as staffing levels and schedules [9, 20] have on Accident and Emergency department performance measures. A more general study analyzed patient time de-

lays in six major hospitals in Dublin [8]. The study identified inappropriate staffing levels of nurses and physicians, confusing medical staff role definitions, long distances to adjacent facilities and inappropriate Accident and Emergency layout structures as the primary causes for patient delays.

While increased waiting time is a problem in Kenya, the phenomenon is worldwide. A five-country hospital survey by [25] found that Canada, Britain and the USA reported average waiting time of two hours or more. R. J Blendon [21], conducted a study in Illinois and found that 84% of the patients had already been examined by a physician an hour after arrival. In Britain, the official land publicized waiting time according to the Patient's Charter is 30 minutes, although the reality may be quite different.

2.2.1 Arrival Rate

This is the rate at which customers arrive at the service facility during a specified period of time. This rate can be estimated from empirical data derived from studying the system or a similar system, or it can be an average of these empirical data. Whenever customers arrive at a rate that exceeds the processing system rate, a waiting line or queue will form. Arrivals may come in singly or in batches; they may come in consistently spaced or in a completely random manner. A potential customer can also leave if, on arrival, he or she finds the line too long. Arrivals at a service are assumed to conform to some probability distribution. On arrival, patients need to be placed in an appropriate queue. Patient flow Management stresses the possibility of segmenting the customers in different queues if appropriate, rather than entering all customers in the same queue. The most common segmentation is based on customer needs, e.g. separate queues for separate services. Customers with more complex service requirements can then be managed separately, which reduces the risk of blocking other customers with a negative impact on their service experience.

For many waiting lines, the arrivals occurring in a given period of time appear to

have a random pattern that is, although we may have a good estimate of the total number of expected arrivals, each arrival is independent of other arrivals, and we cannot predict when it will occur. In such cases, a good description of the arrival pattern is obtained from the Poisson probability distribution:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \text{ where } x = 1, 2, 3, \dots \quad (2.1)$$

where x = number of arrivals in a specific period of time

λ = average number of arrivals for the specific period of time

The arrival process can be characterized by the distribution of the inter-arrival of the customers, denoted by;

$$A(t) = P(\text{interarrival time} < t) \quad [27] \quad (2.2)$$

In queuing theory these inter-arrival times are usually assumed to be independent and identically distributed random variables.

2.2.2 Service Rate

The queuing theory arrivals are described in terms of a rate and service in terms of time. Service times in a queuing process may be any of a large number of different probability distributions. Usually we assume that the service times are independent and identically distributed, and that they are independent-arrival times. For example, the service times can be deterministic or exponentially distributed. It can also occur that service times are dependent of the queue length. The distribution commonly assumed for service times is the negative exponential distribution. The service mechanism describes how the customer is served. It includes the number of servers and the duration of the service time, both of which may vary greatly and in a random fashion. The service time may be similar for each job or it could vary greatly.

In the development of waiting-line models, operations researchers have found that the exponential probability distribution can often be used to describe the service-

time distribution. Its distribution function is denoted by;

$$B(t) = \mu e^{-\mu t} \quad \text{for } t \geq 0 \text{ [27]} \quad (2.3)$$

where; t = service time (expressed in number of time periods)

μ = average or expected number of units that the service facility can handle in a specific period of time.

It is important to use the same time period used for defining arrivals in defining the average service rate. If an exponential service-time distribution is used, the probability of a service being completed within t time periods is given by

$$P(\text{Service time} \leq t \text{ time periods}) = 1 - e^{-\mu t}$$

Many analytical queuing models exist, each based on unique assumptions about the nature of arrivals, service times, and other aspects of the system. Some of the common models are:

Single or multiple channel with Poisson arrivals and exponential service times – this is the most elementary situation.

Single channel with Poisson Arrivals and arbitrary services times – Service times may follow any probability distribution, and only the average and the standard deviation need to be known.

Single channel with Poisson arrivals and deterministic service times – service times are assumed to be constant.

Single or multiple channel with Poisson arrivals, arbitrary service times, and no waiting line – waiting is not permitted, if the server is busy when a unit arrives, the unit must leave the system but may try to re-enter at a later time.

Single or multiple - channel with Poisson arrivals, exponential service times, and a finite calling population – a finite population of units is permitted to arrive for service.

2.2.3 Queue Discipline

A queue discipline is the manner in which new arrivals are ordered or prioritized for service. For the hospital problem, and in general for most customer-oriented waiting lines, the waiting units are ordered on a first-come, first-served (FCFS) basis referred to as an FCFS queue discipline. Other types of queue disciplines are also prevalent. Shortest processing time (SPT), tries to maximize the number of units processed, but units with long processing times must wait long periods of time to be processed, if at all. A random queue discipline provides service to units at random regardless of when they arrived for service. In some cultures, a random queue discipline is used for serving people instead of the FCFS rule.

Triage is used by hospital emergency rooms based on the criticality of the patient's injury as patients arrive. That is, a patient with a broken neck receives top priority over another patient with a cut finger. Preemption is the use of a criterion that allows new arrivals to displace members of the current queue and become the first to receive the service. This criterion could be wealth, society status, age, government position, and so on. Triage is a form of pre-emption based on the patient's degree and severity of medical need. Reservations and appointments allocate a specific amount of capacity at a specific time for a specific customer or processing unit. Legal and medical services, for example, book their day using appointment queuing disciplines. Value-based queuing is a method that allows organizations to prioritize customer calls based on their long-term value to the organization. Low-profitability customers are often encouraged to serve themselves on the company's web site rather than tie up expensive telephone representatives.

A few of these queue disciplines are modeled analytically but most require simulation models to capture system queuing behavior. We will restrict our attention in this chapter to waiting lines with a FCFS queue discipline.

2.2.4 Queue Behavior

People's behavior in queues and service encounters are often different. Reneging is the process of a customer entering the waiting line but later deciding to leave the queue and server system. Balking is the process of a customer evaluating the waiting line and server system and deciding not to enter the queue. In both situations, the customer leaves the system, may not return, and a current sale or all future sales may be lost. Jockeying is the process of customers leaving one waiting line to join another in a multiple-server (channel) configuration.

Most analytical models assume that customers' behavior is patient and steady and they will not renege or balk, as such situations are difficult to model without simulation.

In Systems Theory, a system or a process is in a steady state if the variables that define the behavior of the system remain the same.

2.2.5 Characteristics of Queuing Models

A queuing system can be described as customers arriving for service, waiting for service and leaving the system after being served. A queuing system is characterized by arrival pattern of those requiring service, service pattern of servers, queue discipline, system capacity, number of service channels, and number of service stages. A queuing analysis is based on set of assumptions, namely, that only single individual are coming to a system and that there are no bulk arrivals. Lengths of the intervals between arrivals are independently and identically distributed and described by a continuous density function. It is assumed that inter-arrival times and service times follow the exponential distribution or equivalently that the arrival rate and service rate follow a Poisson distribution. Queue discipline refers to the manner in which waiting patients are selected for service when a queue is formed which could be either first-come first-serve (FCFS) or some other specified priority order.

Different queuing characteristics used include; mean waiting time, incidence of

excessive waiting rather than mean waiting time, average queue length, and expected number of busy and idle servers, probability that those requiring service will not have to wait at all, probability that those needing service may not be served at all, etc. Considering that healthcare is by far most important factor to control, any resource planning in health care context should be based on limiting values of queuing characteristics rather than only average values. With the limiting value it is intended to imply that desired patient waiting times should be zero or near zero, probability that patients will not have to wait should be unity or near one, probability that patients will not be served due to laxity of server should be zero or near zero, expected queue length of patients should be minimal or very small, and expected number of idle servers should not be allowed to increase inordinately.

Conceptually, the simplest queuing model is the single server queue. The system models the flow of customers, wait in the queue if, receive service, and eventually leave. A queuing system consists of customers who have a certain arrival pattern, and are served at a station consisting of a number of servers with a specific service pattern. Queues occur if there is an imbalance between the number of requests for a resource and resource capacity. We use mathematical models to analyze waiting lines. Based on their specific characteristics such as customer arrival, service patterns, and the number of servers available, queues are classified into M/M/1 queuing systems, M/M/s queuing systems, etc. The M/M/1 is the simplest queuing model. In this model, the distribution of arrivals in the system is exponential, the distribution of service time exponential, and the system has a single server.

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2.3 Kendall's Notation

Before starting the investigations of elementary queuing system, let us denote a system by;

$$A/B/m/K/n/D \quad [8] \tag{2.4}$$

where;

A: distribution function of the inter-arrival times,

B: distribution function of service times,

m: number of servers,

K: capacity of the system, the maximum number of customers in the system includ-

ing the one being serviced,

n: population size, number of sources of customers,

D: service discipline.

Exponentially distributed random variables are notated by M, meaning Markovian or memoryless. Hence M/M/1 denotes a system with Poisson arrivals, exponentially distributed service times and a single server. M/G/m denotes an m-server system with Poisson arrivals and generally distributed service times. M/M/r/K/n stands for a system where the customers arrive from a finite source with n-elements where they stay for an exponentially distributed time, the service times are exponentially distributed, the service is carried out according to the requests arrival by r servers and the system capacity K.

2.4 Queuing models

It is common practice in health services to estimate the required number of beds as the average number of daily admissions times average length of stay in days and

Divided by average bed occupancy rate (average number of occupied beds during a day) [5];

$$\text{Bed requirement} = \frac{\text{Average no. of daily admissions}}{\text{Average bed occupancy rate}} \times \text{average length of stay} \quad (2.5)$$

However, as De Bruin A. *et al* [5] mention in the model, only based on average numbers, is not capable of describing the complexity and dynamics of the in-patient flow. Moreover, reported occupancy levels are generally based on the average midnight census (for billing purposes), which results in underestimation of the bed requirements. More recently, queuing models have provided better means of estimating the necessary number of beds based on sound performance measures. The M/G/∞ queue was used as a model for the casualty ward of a hospital [14]. They showed that in steady state, the bed occupancy rate follows a Poisson distribution with mean $\lambda\mu$, where λ denotes the daily admission rate and μ denotes the average duration of stay. Using this model, the authors determine the required number of beds in order

to guarantee that a given target percentage of arrivals receive a bed immediately. Wayne L. W, [27] also used the $M/G/\infty$ system to model the queue of patients needing alternative levels of care in acute care facilities whose treatment is completed and who are waiting to be transferred to an ECF. These patients are kept in the hospital due to unavailability of beds in the ECF and reduce the hospital utilization. The authors' model allows managers to predict the effect of certain policy changes on appropriate access measures. For instance, the cost-benefit trade-off of opening an additional extended care facility within a region is compared to that of assigning a higher priority to patients going to ECF from ACF than to those coming from other sources.

Instead of using an infinite capacity queue, Weiss E and McClain J., [28] used an $M/G/c$ queue with a state-dependent arrival rate to address the long hospital-wait list problem. He experiments with various management actions such as increasing the number of beds or decreasing mean service times through appropriate means. Gans N. *et al* [10] developed a queuing model for bed occupancy management and planning of hospitals. Performance measures, such a Mean bed occupancy and the probability of rejecting an arriving patient due to hospital overcrowding, are computed. These quantities enable hospital managers to determine the number of beds needed in order to keep the fraction of delays under a threshold, and also to optimize the average cost per day by balancing the costs of empty beds against those of delayed patients. Although service times, unlike inter- arrival times, do not usually have an exponential distribution, such an assumption is often made in order to simplify the analysis greatly. For instance, De Bruin A. *et al* [5] used the $M/M/c/c$ queue, referred to as the Erlang Loss model, to investigate the emergency in-patient of cardiac patients in a university medical Centre in order to determine the optimal bed allocation so as to keep the fraction of refused admissions under a target limit. The authors find the relation between the size of a hospital unit, occupancy rate, and TAR, cancellation rate of 5. Another queuing network model applied to a hospital

setting is that of Manuel, Laguna [18], who studied a specific obstetrics hospital consisting of 8 sub units with 4 different patient arrival streams. The transfer of patients between the different compartments creates delay in some of the units. As can be seen, the application of queuing models to health care is growing more popular as hospital management teams are gaining awareness of the advantages of these operational research techniques in addressing such issues as determining optimal bed counts and making policy decisions with regards to resource allocation. Research in applying queuing networks with blocking is rarer in the literature due to the mathematical complexities involved in computing performance measures associated with such systems. As a result, hospitals with interacting subunits are often studied through simulations, for they are able to incorporate much more detail than is affordable by analytical methods.

2.5 Practical Background: The ED - IW Process

2.5.1 Hospital, ED and IWs

MTRH is the second largest referral hospital in Eldoret, Kenya with approximately 84,000 patients hospitalized yearly. It is composed of several departments such as oncology, dental, ED, nursing among others. The ED has an average admission rate of 124 patients daily and a capacity of 1000 beds; and 28 IWs. It is divided into several subunits namely: Ambulatory, CAR-E, Huduma Centre, Minor Theatre, Routine Lab, Emergency X-Ray, CSSD, Plaster and Surgical Rooms. The wards are classified as emergency, surgical and pediatrics. Refer to table 3.2 detailing the specific IWs and MUs within the facility.

The ED can be described as the reception of the MTRH because 99% of the arriving patients are received, diagnosed and discharged or admitted from the department. The other departments handle specialized treatment such as cancer, dental among others. The research therefore concentrates on modeling and analyzing congestion at the ED.

2.5.2 The Routing Process

The working process in the ED: the first patient-personnel interaction occurs in the triage area, where a triage nurse evaluates the patient, determines the seriousness of the patient's health condition and assigns a corresponding triage level or code which ranges from 1 to 3 as can be seen in table 3.1. Patients with level 1 proceed directly to the resuscitation room and those with level 2 to the immediate care unit. Patients with levels 3, move to the waiting area unless a bed is immediately available.

Table 2.1: Triage levels (triage codes) in ED

Triage level/Code	Waiting time to first medical consult
Level I / Red code	30 minutes
Level II/ Yellow code	Maxim 45 minutes
Level III/ Green code	Maxim 60 minutes

The workflow of the ED will be analyzed as an absolute queuing process, the cases with red code are treated by priority, in which the patients arrive, wait, are evaluated and treated and then they are discharged or are transferred in the hospital units. Triage is a form of preemption used by hospital emergency rooms based on the criticality of the patient's as patients arrive. For example, a patient with a broken neck receives top priority over another patient with cut finger.

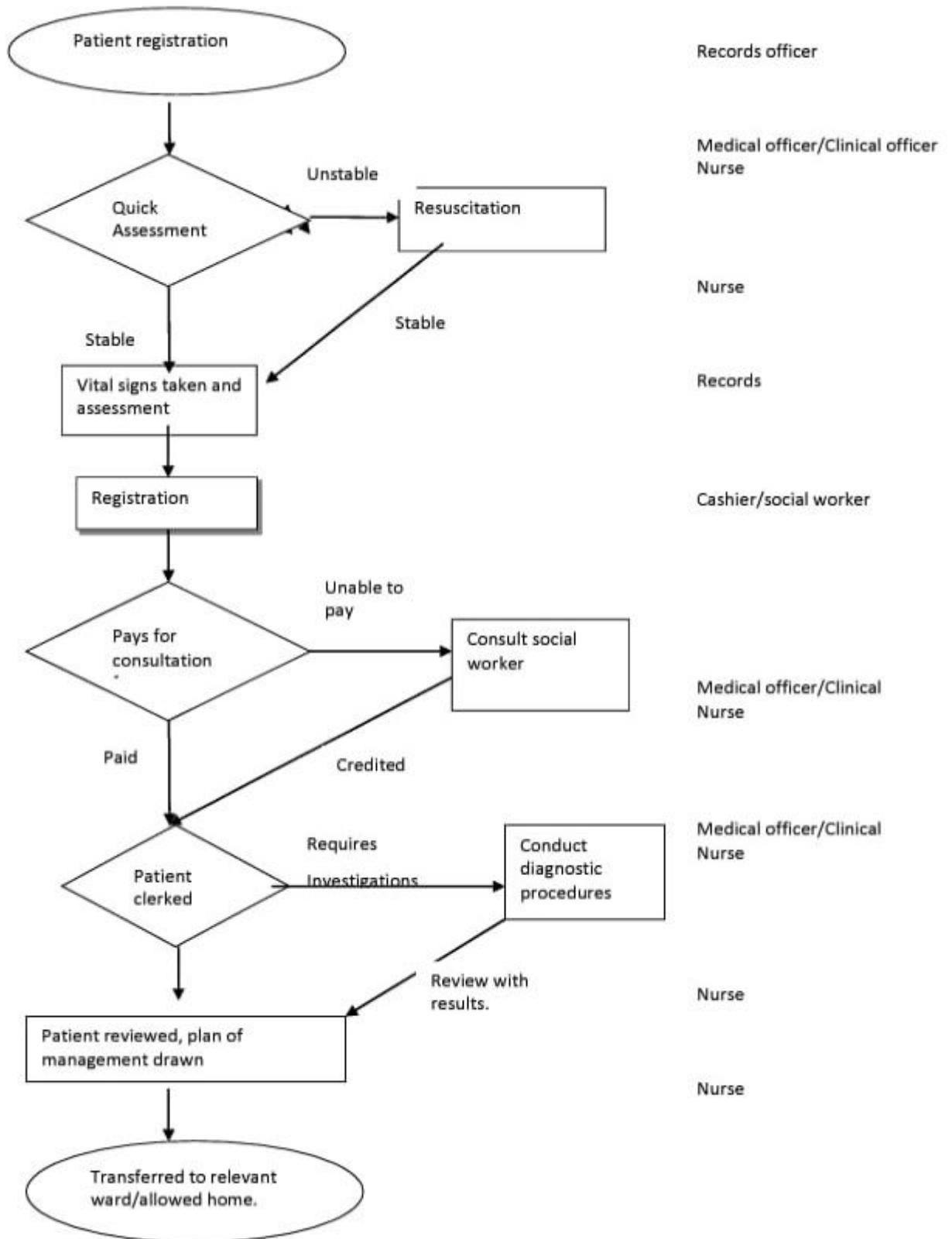


Figure 2.1: ED-IW Integrated Activities/Resource Flow Chart Diagram

2.6 Wards Operating Characteristics

Although most of the wards provide similar medical services, they do differ in their operational measures as elaborated below. First, each medical unit is characterized by its capacity. Ward's capacity is measured by its number of beds (static capacity) and number of service providers - doctors, nurses, administrative staff and general workers (dynamic capacity). Generally (and in our hospital IWs in particular), the latter is determined proportionally to the former (see, however, discussions on the appropriateness of such proportional" staying in [19, 25, 2]). For example, an IW nurse-to-bed ratio is one to three whereas nurse-to- patient ratio is one to six. Hence a unit's operational capacity can be characterized by the number of its beds only - denoted as its standard capacity. Maximal capacity stands for standard capacity plus extra beds that can be placed in corridors in overloaded periods.

In addition, medical units can be characterized by various performance measures: operational - average bed occupancy level, ALOS, waiting times for various resources, number of patients hospitalized per bed per time unit (flux); and quality - patients' return rate, patients' satisfaction, mortality rate, etc. Note that occupancy rate and flux are calculated relative to wards standard capacities. Comparing two basic measures, ward capacity and ALOS, we observed that the wards differ on both. It is worth noting that the ALOS is dependent on the nature of medical service provided in each IW or unit, for instance, the Isolation Ward, one of the smallest wards in terms of bed capacity has the highest ALOS despite a single admission during the monthly average of the indicated period.

Table 2.2: Internal Wards Operating Measures

Wards	Allocated Beds	Total Admissions	Mortality Rate (%)	Average Occupancy (%)	Alos (Days)	Discharges	Number of Nurses
Amani	92	253	51(20%)	93.66	11.00	262	33
Umoja	96	269	54(20%)	87.33	9.67	274	32
CCU	14	44	9(20%)	12.33	8.33	44	22
Isolation Ward	5	1	1(100%)	3.67	67.67	1	4
Faraja	40	236	8(3%)	52.67	6.67	243	18
Riley Mother & Baby	125	1161	3(0.3%)	28.67	1.00	1090	135
Neema	60	256	49(19%)	96.67	11.33	261	*****

Wards	Allocated Beds	Total Admissions	Mortality Rate (%)	Average Occupancy (%)	Alos (Days)	Discharges	Number of Nurses
Upendo	32	143	10(7%)	45.33	9.33	145	17
Tumaini	35	138	14(10%)	49.33	10.67	143	17
Subira	35	138	6(4%)	51.33	11.33	142	18
Fadhili	27	156	2(1%)	42.67	8.00	160	15
Burns Peads	22	21	1(5%)	22.67	37.33	19	*****
Neuro Peads	11	28	2(7%)	12.67	12.33	31	16
Neuro (Male)	20	88	8(9%)	37.67	11.33	102	24
Neuro (Female)	10	25	3(12%)	10.00	11.67	27	*****
Longonot	35	121	1(0.8%)	58.67	13.33	137	22
Sergoit	12	49	1(2%)	19.00	12.00	49	14
Kilimanjaro	49	163	12(7%)	65.33	11.67	171	31
Rehema	46	97	11(11%)	33.33	10.33	98	23
ICU	21	96	41(43%)	19.00	6.33	95	58
HDU	3	3	0(0%)	0.33	1.33	3	*****
Elgon	24	60	0(0%)	10.33	5.33	61	14
Kenya	80	97	1(1%)	81.00	23.00	110	19
Ada	16	4	0(0%)	15.33	100.00	6	9
Pw I	51	156	10(6%)	40.67	7.67	161	18
Pw II Adult	38	125	7(6%)	32.67	7.67	127	34
Pw II Peads	6	57	1(2%)	10.00	5.33	57	*****
Pw II Mat	12	65	0(0%)	6.67	3.00	67	*****
Pw II Nbu	3	2	1(50%)	0.33	6.33	2	*****
Total	1020	4054	307(8%)		441	4088	593

*** Data relates to the monthly average for the period 1/1/2020 - 30/3/2020 ***

2.6.1 Problems in the ED-to-IW Process

Two main problems identified in the process of patients routing from the ED to the IWs are long delays to admission and fairness, described and analyzed as follows .

Long delays to admission

Once the decision to admit a patient is made, the next stage of the negotiations is agreeing upon the time at which the patient will be transferred to his/her ward. Here interests are conflicting: the ED seeks to discharge the patient as soon as possible in order to be able to accept new ones, and the IWs wish to have the move carried out at a time convenient for them. From conversations with nurses from both sides we learn that, when deciding on a patient's transfer time, the main issue taken into account (assuming there is an available bed in the ward) is nurses' and doctors' availability (they might be unavailable because of treating other patients, shifts changing or meals, various staff meetings or resuscitation). Another parameter is the availability of necessary equipment and other logistic considerations: for example, preparation for a complicated" patient, who requires special bed/equipment, or placement near

a nursing station, takes a longer time.

Patients to be hospitalized wait in the ED till transfer to their ward is carried out - sometimes these waiting times are extremely long. Through the Causes and Effects Chart (Fish-bone Diagram) in Figure 3.2 one sees the various causes of these long delays. We wish to emphasize that the delays are caused not only by beds unavailability: patients usually wait even when there are available beds.

Long waiting times cause an overload on the ED, as beds remain occupied while new patients continue to arrive. Improving the efficiency of patients flow from the ED to the IWs, while shortening waiting times in the ED, will improve the service and treatment provided to patients. In addition, reducing the load on the ED will lead to a better response to arriving patients and it is likely to save lives.

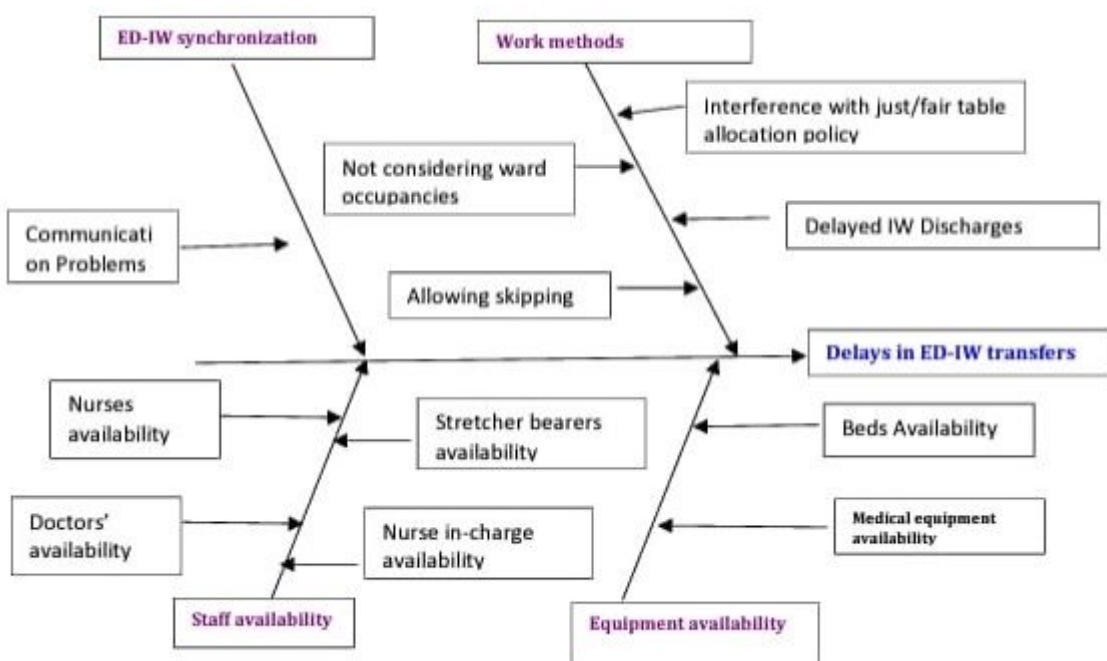


Figure 2.2: ED-IW Delays: Causes and Effects Chart

Fairness

Each nurse/doctor should have the same workload i.e each nurse/doctor should take care, at any given time, of an equal number of patients. As the number of nurses and doctors is usually proportional to standard capacity, this criterion is equivalent

to keeping beds occupancy rates equal among the wards. However, if one maintains occupancy levels equal then, by Little's law, wards with shorter ALOS will have a higher turnover rate (discharges) and thus admit more patients per bed. But the load on the ward staff is not spread uniformly over a patient's stay, as treatment during the first days of hospitalization requires much more time and effort from the staff than in following days [22]; in addition, patients' admissions and discharges consume doctors' and nurses' time and effort as well. Thus, even if the occupancy among wards is kept equal, the ward admitting more patients per bed ends up having a larger load on its staff. Hence, a natural alternative fairness criterion is balancing the incoming load, or flux - namely, the number of admitted patients per bed per certain time unit among the wards. We examine the proposed fairness criteria in the process with the help of the above table.

Considering NEEMA Ward, which is among the smallest (bed capacity) and the fastest (short ALOS). We observe that the average occupancy rate in this ward is high. In addition, the number of patients hospitalized per month in this ward equals about 6.29% of the number of patients hospitalized per month in the other wards, although its size is just about 2/3 of the others. And indeed, the flux in FADHILI (6 patients per bed per month) is significantly higher than in the other wards, hence (from the discussion above) the load on its staff is the highest. Short ALOS could result from a superior efficient clinical treatment; but it might alternatively be a consequence of too-early discharge of patients, which may also be undesirable.

We see that the patients routing does not appear to be fair either towards staff, or towards patients. Increasing fairness in the routing process will increase staff satisfaction, provide incentives for improved care and cooperation (for the importance of service providers' equity perception). This will also improve patients' satisfaction, in particular their perception of the quality of care.

CHAPTER 3

MODEL FORMULATION

3.1 Introduction

In this chapter, a queuing model is formulated. The model is developed based on key ward operational measures such as admissions/arrivals, ALOS which represent the service rates in the various wards and standard bed capacity. The chapter begins by providing brief literature on modeling patient flow in any health care facility, stating the underlying assumptions, defining variables and parameters and thereafter developing the model.

3.2 Model of patient flow

It is common for health care managers to project workload for physical infrastructure and manpower planning. They must consider five typical measures when evaluating the existing or proposed service systems. These measures are:

- average number of patients waiting (in queue or in the system);
- average time the patients wait (in queue or in the system);
- capacity utilization
- costs of a given level of capacity;
- Probability that an arriving patient will have to wait for service.

The system utilization measure reflects the extent to which the servers are busy rather than idle. On the surface, it might seem that health care managers would seek 100% system utilization. Under normal circumstances, 100% utilization may not be realistic; a health care manager should try to achieve a system that minimizes the sum of waiting costs and capacity costs. In queue modeling, the health care manager

also must ensure that the average arrival and the service rates are stable, indicating that the system is in a steady state, a fundamental assumption. The main queuing model characteristics are (Yasara, 2009):

- the population source;
- the number of servers;
- the arrival patterns and the service patterns;
- the queue discipline.

The population source can be infinite or finite. In an infinite source situation, patient arrivals are unrestricted, and can exceed the system's capacity at any time.

Number of servers - the capacity of the queuing systems is determined by the capacity of each server and the number of servers being used.

Arrival patterns – the waiting lines occur because highly variable arrivals and service patterns cause the systems to be temporarily overloaded. The hospital ED is very good examples of random arrival patterns causing such variability. The arrival pattern is different at different times of the day.

Service patterns - because of the varying nature of the illnesses and the patients' conditions, the time required for treatment varies from patient to patient.

Queue discipline refers to the order in which customers are processed. The assumption that service is provided on a first-come, first-served (FCFS) basis is the most commonly encountered rule. The ED does not serve on this basis, patients do not all represent the same risk, level of triage; those with the highest risk, the most seriously ill, are treated first.

The queue system is usually described in shorten form by using some characteristics. These characteristics can be represented by Kendall's notation which was initially a three factor notation A/B/C. Later D, E and F were also included in the model to make it A/B/C/D/E/F. The notation of the queuing model is presented in Table 4.1; Examples of some special notations for various probability distributions

Table 3.1: Kendall's queuing model Notation

Symbol	Explanation
A	The arrival time distribution
B	The service time distribution
C	The number of servers (agent available)
D	The system's capacity, the number of customers in the system
E	The calling population
F	The queue discipline

describing arrivals and departures include: M - Arrival or departure distribution that is a Poisson process, E - Erlang distribution, G - General distribution, GI - General independent distribution.

For the application of the queuing models to any situation we should describe first the input and the output process. In our ED the input process is the patient's arrival and the output process is considered the patient's discharge in the hospital unit.

For the application of the queuing models to any situation we should describe first the input and the output process. In our ED the input process is the patient's arrival and the output process is considered the patient's discharge in the hospital unit.

3.3 Assumptions and variables

The following assumptions are made in the development of the model:

1. Arrivals occur one by one in a Poisson stream with mean arrival rate $\lambda(> 0)$
2. Arrivals are from an infinite source population.
3. The system has heterogeneous server pools representing the wards.
4. Each ward contains a number of i.i.d servers corresponding to the number of beds.
5. Service is provided on a FCFS basis (arrivals wait in a single line and then move to the first open server for service)

6. Service times are exponentially distributed with mean service time μ which is the same for each server.
7. Work-conserving (there are no idle servers whenever there are delayed customers in the queue)
8. No balking (evaluating and deciding not to join a queue) or reneging (opting out of a queue).
9. Factors like staff, medicine, infrastructural requirements are held constant.

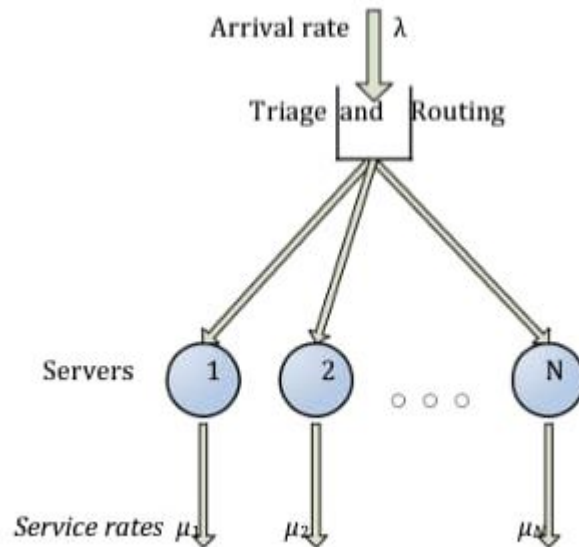


Figure 3.1: Queuing System with Heterogeneous Servers

In order to build the flowchart of the patients' accessing and departing the ED we will use the queuing theory to determine the minimal number of servers (providers) needed. Queue models generally deal with customer arrivals at a service facility. We will consider an M/M/k queuing model because it will help us to estimate the number of providers needed. Arrivals occur according to a Poisson process and the service duration has an exponential distribution. Poisson distribution is a discrete

distribution that shows the probability of arrivals in a given time period, where the mean and the variance of the Poisson distribution are the same.

3.4 Ward Operating Measure/Parameters

Using this M/M/k model, it is known that the system is in a steady state if the following relation is fulfilled:

$$\frac{\lambda}{k\mu} < 1 \quad (3.1)$$

We denote the parameters by:

k = number of servers/channels/wards

λ = mean arrival rate for the system

μ = mean service rate for each channel/ward = ALOS

P_0 = probability of zero units in the system

P_n = probability of n units in the system

L_q = the average number of units waiting for service

L = the average number of units in the system or queue length

W_q = the average time a unit spends waiting for service

W_t = service time = $\frac{1}{\mu}$

W = the average time a unit spends in the system

P_w = the probability that an arriving unit must wait for service

$\frac{\lambda}{\mu}$ = utilization factor of the system

3.5 The Queuing Model Formulation

To optimize the process, we are looking for the probability P_k the probability that an entering patient must queue for treatment which means that all beds are busy. In order to calculate these probabilities, we will use relations (1).

We denote λ_{wi} as the monthly average number of arrivals of the patients in i^{th} ward in the time interval $[t_{i-1}, t_i]$, where $t_0=0$ means first day of the month. On a

given month in the time interval $[t_{i-1}, t_i]$ per ward, then the daily average number of the patient's arrival on the queue/day we obtain from: $\lambda = (\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_{28})/30 = \sum_{i=1}^{28} \frac{\lambda_i}{30}$, where $i = 1, 2, 3, \dots, 28th$ ward. Thus, the average number of daily arrivals for the system denoted by lambda will be:

$$\lambda = \sum_i^{28} \frac{\lambda_i}{30} = \frac{4053.6664}{30} = 135.1222 \text{ patients/day}$$

Thus we will consider in the following that patients arrive at a rate of 135.1222 patients/day and stay in a single queue. Using the created database and same rationality as above, the calculated daily service rate is: $\mu = (Daily\ discharges \times ALOS)/30$ for the respective wards which in most instances are unique.

It is known that the system is in steady state if the relation is fulfilled: $\frac{\lambda}{k\mu} < 1$ where k presents the number of servers/channels in the ED. Thus, using the inequality can be estimated the minimum number of servers in the ED.

We used M/M/k queuing model to estimate different specifications of the queue, different characteristics of the ED. In our case study we have: $\lambda = 135.1222$; $\mu = (Daily\ discharges \times ALOS)/30$; servers = k; utilization factor = $\frac{\lambda}{\mu}$. In the case of M/M/k model to calculate the probability that no patients is in the ED, we use the condition, the overall sum of probabilities must be 1. We write:

$$P_0 + P_1 + P_2 + P_3 + \dots + p_k = 1 \quad (3.2)$$

We substitute the probabilities for k servers and we get:

$$P_0 + P_0 \left(\frac{\lambda}{\mu} \right) + P_0 \left(\frac{(\frac{\lambda}{\mu})^2}{2!} \right) + \dots + P_0 \left(\frac{(\frac{\lambda}{\mu})^{k-1}}{(k-1)!} \right) + P_0 \left(\frac{(\frac{\lambda}{\mu})^k}{k!} \frac{k\mu}{(k\mu - \lambda)} \right) = 1 \quad (3.3)$$

$$P_0 \left[1 + \frac{\lambda}{\mu} + \left(\frac{(\frac{\lambda}{\mu})^2}{2!} \right) + \dots + \left(\frac{(\frac{\lambda}{\mu})^{k-1}}{(k-1)!} \right) + \left(\frac{(\frac{\lambda}{\mu})^k}{k!} \frac{k\mu}{(k\mu - \lambda)} \right) \right] = 1 \quad (3.4)$$

$$P_0 = \frac{1}{\left[1 + \frac{\lambda}{\mu} + \left(\frac{(\frac{\lambda}{\mu})^2}{2!} \right) + \dots + \left(\frac{(\frac{\lambda}{\mu})^{k-1}}{(k-1)!} \right) + \left(\frac{(\frac{\lambda}{\mu})^k}{k!} \frac{k\mu}{(k\mu - \lambda)} \right) \right]} \quad (3.5)$$

The geometric series is convergent and introducing the sum of the series, we have:

$$P_0 = \left[\sum_{i=0}^{k-1} \frac{\left(\frac{\lambda}{\mu}\right)^i}{i!} + \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!} \frac{k\mu}{(k\mu - \lambda)} \right]^{-1} \quad (3.6)$$

Therefore, the probability that no patient is in the ED is as stated in (3.6) above.

In M/M/n queue an entering patient must queue for service exactly when n or more patients are already in the system. Using Erlang's C formula, we get in our case:

$$\begin{aligned} P_w &= \frac{\frac{A^N}{N!} \frac{N}{N-A}}{\left(\sum_{i=0}^{N-1} \frac{A^i}{i!} \right) + \frac{A^N}{N!} \frac{N}{N-A}} \\ &= \sum_{k=29}^{\infty} p_k = 1 - \sum_{k=0}^{28} p_k \\ P_w &= p_0 \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!} \frac{k\mu}{(k\mu - \lambda)} \end{aligned} \quad (3.7)$$

Equation (3.7) is the probability that an arriving patient must wait for service, denoted by p_w . The length of the queue in the case of M/M/n is given by:

$$L_q = p_0 \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!} \sum k \left(\frac{\lambda}{k\mu}\right)^k \quad (3.8)$$

Calculating the sum of the series we obtain

$$L_q = p_0 \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!} \frac{k\mu\lambda}{(k\mu - \lambda)^2} \quad (3.9)$$

Simplifying, we get

$$L_q = p_0 \frac{\left(\frac{\lambda}{\mu}\right)^k}{(k-1)!} \frac{\mu\lambda}{(k\mu - \lambda)^2} \quad (3.10)$$

Equation (3.10) is the number of patients in the queue or simply the queue length, denoted by L_q .

Now we were interested to knowing the waiting time in queue for service. Using the Little's Law, the mean waiting time in the queue can be obtained from:

$$W_q = \frac{L_q}{\lambda} \quad (3.11)$$

The average treatment/service time per patient, denoted by W_t is

$$W_t = \frac{1}{\mu} \quad (3.12)$$

Thus the total waiting time in system can be obtained from:

$$W = W_q + W_t \quad (3.13)$$

The overall number of patients in the ED is an average

$$L = W\lambda \quad (3.14)$$

3.6 Bed Requirement Approximation Model

It is common practice in health services to estimate the required number of beds as the average number of daily admissions times average length of stay in days and divided by average bed occupancy rate (average number of occupied beds during a day);

$$\text{Bed requirement} = \frac{\text{Average no. of daily admissions}}{\text{Average bed occupancy rate}} \times \text{average length of stay} \quad (3.15)$$

However, as de Bruin *et al.* [5] mention in, model, only based on average numbers, is not capable of describing the complexity and dynamics of the in-patient flow. Moreover, reported occupancy levels are generally based on the average midnight census (for billing purposes), which results in underestimation of the bed requirements. More recently, queuing models have provided better means of estimating the necessary number of beds based on sound performance measures. In Pike *et al.* use the M/G/ ∞ queue as a model for the casualty ward of a hospital. They show that in steady state, the bed occupancy rate follows a Poisson distribution with mean $\lambda\mu$, where λ denotes the admission rate (arrival rate) and μ denotes the average duration of stay per ward. Using this model, the authors determine the required number of beds in order to guarantee that a given target percentage of arrivals receive a bed immediately.

N/B: Overall, the facility requires approximately 5,410 beds broken down per ward/unit as indicated in the table above. Some bed requirement figures are unnecessarily too high; depending on how huge the ALOS and how small the denominator (Average daily occupancy) are. This formula seems inaccurate as even small capacity wards with average ALOS are reporting outrageous bed requirements which is unnecessary. As a result, an improvement of the formula is required. This is supported by the fact that the denominator (Average daily occupancy) is a factor of the ALOS. We wish to improve the formula as follows:

$$\textit{Bed requirement} = \textit{Average daily admissions} \times \textit{Average Length of Stay} \tag{3.16}$$

CHAPTER 4

RESULTS

4.1 Introduction

This chapter entails, applying the model to analyze data collected. The analysis also entails computing bed requirement per ward which guarantees optimum access to health care for in-patients as well as the wards' operating characteristics/measures.

4.2 Operating Characteristics Results of Queuing model

Equation (3.14) is related to the known $Distance = time \times speed$. Referring to Table 4.1 summarizing the various wards/units performance measures, the following equations apply to multiple-server waiting lines for which the overall mean service rate, $k\mu$, is greater than the mean arrival rate, λ . In such cases, the service rate is sufficient to process all arrivals where; The ratio $\frac{\lambda}{\mu}$ is often referred to as the utilization factor for the waiting line, the probability that all k service channels are idle (that is, the probability of zero units in the system):

$$P_0 = \frac{1}{\left[\sum_{i=0}^{k-1} \frac{(\frac{\lambda}{\mu})^i}{i!} + \frac{(\frac{\lambda}{\mu})^k}{k!} \frac{k\mu}{(k\mu - \lambda)} \right]}$$

, the probability of n units in the system:

$$p_n = \frac{(\frac{\lambda}{\mu})^n p_0}{k! k^{n-k}}, \text{ for } n > k$$
$$p_n = \frac{(\frac{\lambda}{\mu})^n p_0}{n!}, \text{ for } n \leq k$$

the average number of units waiting for service:

$$L_q = p_0 \frac{(\frac{\lambda}{\mu})^k}{(k-1)!} \frac{\mu\lambda}{(k\mu - \lambda)^2}$$

the average number of units in the system:

$$L = L_q + \frac{\lambda}{\mu}$$

The same equation can also be given by $L = W\lambda$ the average time a unit spends waiting for service:

$$W_q = \frac{L_q}{\lambda}$$

the average time a unit spends in the system (waiting time plus service time):

$$W = W_q + \frac{1}{\mu}$$

the probability that an arriving unit must wait for service:

$$P_w = p_0 \frac{\left(\frac{\lambda}{\mu}\right)^k}{(k-1)!} \frac{\mu}{(k\mu - \lambda)}$$

The summary of operating characteristics was as shown in table 4.1 which summarizes the operating characteristics of the multi-server ED- IW.

Table 4.1: Summary operating characteristics of the multi-server ED- IW

Wards	$\rho = \frac{\lambda}{k\mu}$	P_0	L_q	L	W_q	W	P_w
Amani	0.0503	0.2445	$6.5305e^{-01}$	$1.4084e^{+01}$	$4.8331e^{-03}$	0.0104	$9.3076e^{-01}$
Umoja	0.0547	0.2164	$6.4961e^{-01}$	$1.5305e^{+01}$	$4.8076e^{-03}$	0.0113	$9.7771e^{-01}$
CCU	0.3919	$1.7170e^{-05}$	$8.0197e^{-06}$	10.9724	$5.9352e^{-03}$	0.0812	$1.0000e^{+00}$
Isolation Ward	3.2093	-	-	-	-	-	-
Faraja	0.0895	0.0816	$4.2743e^{+00}$	$2.5057e^{+01}$	$3.1633e^{-02}$	0.0185	$9.5649e^{-01}$
Riley Mother & Baby	0.1329	0.0242	$1.3274e^{+00}$	$1.7201e^{+01}$	$9.8234e^{-03}$	0.0275	$9.7992e^{-01}$
Neema	0.0489	0.2544	$2.9539e^{-01}$	$1.3687e^{+01}$	$2.1861e^{-03}$	0.0101	$9.5886e^{-01}$
Upendo	0.1067	0.0504	$4.5374e^{+00}$	$2.9885e^{+01}$	$3.3580e^{-03}$	0.0221	$9.8108e^{-01}$
Tumaini	0.0949	0.0701	$2.0473e^{-01}$	$1.6576e^{+01}$	$1.5151e^{-03}$	0.0197	$8.7786e^{-01}$
Subira	0.0902	0.0801	$5.2329e^{-01}$	$2.5248e^{+01}$	$3.8727e^{-03}$	0.0187	$7.9149e^{-01}$
Fadhili	0.1131	0.0421	$2.0705e^{+00}$	$1.1669e^{+01}$	$1.5323e^{-02}$	0.0234	$9.9381e^{-01}$
Burns Peads	0.2077	0.0030	$8.3296e^{+00}$	$2.8168e^{+01}$	$6.1645e^{-14}$	0.0430	$2.0819e^{-08}$
Neuro Peads	0.3746	$2.7823e^{-05}$	$3.3333e^{+00}$	10.4896	$2.4669e^{-03}$	0.0776	$2.0066e^{-01}$
Neuro (Male)	0.1256	0.0297	$3.1657e^{-01}$	$1.5181e^{+01}$	$2.3428e^{-03}$	0.0260	$8.3724e^{-01}$
Neuro (Female)	0.4653	$2.1944e^{-06}$	0.0002	13.0298	$1.4326e^{-03}$	0.0964	0.0646
Longonot	0.0791	0.1093	$1.5406e^{-01}$	$2.2138e^{+01}$	$1.1404e^{-03}$	0.0164	$8.0673e^{-01}$
Sergoit	0.2462	0.0010	$4.3235e^{-01}$	$1.8940e^{+01}$	$3.1997e^{-03}$	0.0510	$7.1646e^{-01}$
Kilimanjaro	0.0724	0.1316	$1.4388e^{-01}$	$2.0280e^{+01}$	$1.0648e^{-03}$	0.0150	$7.4375e^{-01}$
Rehema	0.1430	0.0183	$8.5760e^{-01}$	$1.4030e^{+01}$	$6.3469e^{-03}$	0.0296	$9.8697e^{-01}$
ICU	0.2415	0.0012	$2.7731e^{+00}$	$2.7611e^{+01}$	$2.0523e^{-02}$	0.0500	$9.0520e^{-01}$
HDU	32.5847	-	-	-	-	-	-
Elgon	0.4450	$3.8784e^{-06}$	$8.6873e^{-02}$	12.4602	$6.4292e^{-02}$	0.0922	$9.0329e^{-01}$
Kenya	0.0572	0.2014	$2.2969e^{-09}$	5.6022	$1.6999e^{-11}$	0.0119	$8.9337e^{-02}$
Ada	0.2555	0.0008	$9.9838e^{-10}$	7.1535	$7.3887e^{-12}$	0.0529	$1.5740e^{-06}$
Pw I	0.1170	0.0377	$5.0526e^{+00}$	$1.5773e^{+01}$	$3.7393e^{-02}$	0.0243	$4.3964e^{-01}$
Pw II Adult	0.1491	0.0154	$2.4709e^{+00}$	$1.9743e^{+01}$	$1.8286e^{+01}$	0.0309	$9.2794e^{-01}$
Pw II Peads	0.4735	$1.7482e^{-06}$	0.0003	13.2572	$1.9432e^{-06}$	0.0981	$9.0833e^{-01}$
Pw II Mat	0.7167	$1.9041e^{-09}$	0.1646	20.2323	0.0012	0.1497	$1.0000e^{+00}$
Pw II Nbu	9.7966	-	-	-	-	-	-

Statistics relate to the period 1/1/2020 - 31/3/2020

The results showed that the probability that no patient in the ED, no queues is: $P_0 = 0.2445$ for Amani, 0.2164 for Umoja, $1.7170e^{-05}$ for CCU and unstable for

Isolation Ward because $\frac{\lambda}{k\mu} > 1$ which indicated that the system would collapse if the rate of admission exceeded the number of patients who were being isolated.

Applying Equation (3.7) in our data we get, $P_w = 9.3076e^{-01}$, $9.7771e^{-01}$, $1.0000e^{+00}$ for Amani, Umoja and CCU respectively among others. Also, applying equation (3.10), we got the following queue lengths; Amani = 0.6531, Umoja = 0.6496, CCU = 0.8020 among others. The total waiting time (W) for Amani was 0.0104 days (0.2496 hours), for Umoja we had 0.0113 days (0.2712 hours) and CCU had a total waiting time equal to 0.0812 days (1.95 hours) among others. Therefore from the data the overall number of patients in the system will be: L = 14.084 in Amani, 15.305 in Umoja and 10.9724 in CCU among other wards.

4.3 Analysis of Bed Requirement Approximation Model

Applying the above formulas in equations 3.15 and 3.16 on the data, we get the results of bed requirement for improved formula.

Table 4.2: Bed Requirement for both unimproved and improved formula

Wards	Current Bed Allocation	Average Daily Admissions	Average Daily Occupancy	ALOS	Bed Requirement	Bed Requirement for Improved Formula
Amani	92	8.4444	2.8724	11.000	32.3383	92.8884
Umoja	96	8.9667	2.7947	9.6667	31.0154	86.6784
CCU	14	1.4667	0.0576	8.3333	212.1953	12.2225
Isolation Ward	5	0.0222	0.0061	67.6667	246.2624	1.5022
Faraja	40	7.8778	0.7022	6.6667	74.7920	52.5189
Riley Mother & Baby	125	38.7111	1.1944	1.0000	32.4105	38.7111
Neema	60	8.5222	1.9333	11.3333	49.9584	96.5846
Upendo	32	4.7778	0.4836	9.3333	92.1336	44.5926
Tumaini	35	4.5889	0.5756	10.6667	85.0390	48.9484
Subira	35	4.6111	0.5989	11.3333	87.2583	52.2590
Fadhili	27	5.2111	0.3840	8.0000	108.5646	41.6888
Burns Peads	22	0.7000	0.1662	37.3333	157.2401	26.1333
Neuro Peads	11	0.9222	0.0464	12.3333	245.1243	11.3738
Neuro (Male)	20	2.9444	0.2511	11.3333	132.8943	33.3698
Neuro (Female)	10	0.8444	0.0333	11.6667	295.8367	9.8514
Longonot	35	4.0333	0.6844	13.3333	78.5757	53.7772
Sergoit	12	1.6222	0.0760	12.0000	256.1368	19.4664
Kilimanjaro	49	5.4333	1.0671	11.6667	59.4028	63.3887
Rehema	46	3.2333	0.5111	10.3333	65.3701	33.4107
ICU	21	3.1889	0.1330	6.3333	151.8516	20.1963
HDU	3	0.1111	0.0003	1.3333	493.7654	0.1481
Elgon	24	2.0000	0.0827	5.3333	128.9794	10.6667
Kenya	80	3.2444	2.1600	23.0000	34.5469	74.6212
Ada	16	0.1333	0.0818	100.0000	162.9584	13.3300
Pw I	51	5.2111	0.6913	7.6667	57.7925	39.9519

Wards	Current Bed Allocation	Average Daily Admissions	Average Daily Occupancy	ALOS	Bed Requirement	Bed Requirement for Improved Formula
Pw II Adult	38	4.1667	0.4138	7.6667	77.1987	31.9448
Pw II Peads	6	1.9000	0.0200	5.3333	506.6635	10.1333
Pw II Mat	12	2.1667	0.0268	3.0000	243.4494	6.5001
Pw II Nbu	3	0.0667	0.0003	6.3333	1408.1037	0.4224
Total	1020	135.1222			5607.8580	945.8555

*** Data relates to the period 1/1/2020 - 30/3/2020 ***

The number of bed requirement generated in the above table for the new formula are now within the bed capacity range set by the Hospital. This criterion is fair, efficient, cost effective and an equitable redistribution or optimum allocation of beds among the wards and units.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The chapter briefly summarizes the research by making conclusion, recommendations and future research work related to or arising from the study.

5.2 Conclusions

We have described and analyzed the congestion scenario at the ED and IW of MTRH as set out in the research objectives by modelling and computing bed requirements for the IWs/units, and analyzing other operating characteristics. These measures should assist the management to ease congestion in the facility.

5.3 Recommendations

Complete eradication of queues in any service facility remains a challenge given the fact that resources (servers) are ever limited despite high demand for such services. However, for an effective management of a health facility, the negative effects can be mitigated by close monitoring of key ward performance measures. For wards' staff-fairness to be achieved, the management should strive to balance the occupancy rate and the flux (number of patients per bed per unit time) of the wards. Developing an accurate and consolidated patient service computerized template will aid in recording and managing these key characteristics in the ED as well as the whole facility.

5.4 Future Research

It is proposed that related future study should focus on the following:

1. Incorporate and analyze outpatient scenario of the facility: the study focuses on bed requirements which relates mainly to the inpatient. Future studies

should include the outpatient so as to capture the picture of congestion at the entire facility and not just at the wards.

2. Incorporate game theory/analysis which involves efficiency and cost saving approach – the economics of waiting.
3. Statistics per cadre of patients to be included. The data collected and analyzed are general; a breakdown of the figures by the three categories of patients (Red, Yellow and Green) should be clearly captured and analyzed to adequately address the issue of fairness in ward admission.
4. Carry out patients' satisfaction surveys in terms of return rate, treatment quality and related psychological studies.
5. Develop an algorithm for routing patients to the various ED – IW so as to minimize any possibility human bias/error by the triage nurse.
6. Do simulation Modelling to compare and contrast simulated and theoretical results.

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APPENDIX I: Approval – IERC MMUST



MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY
P. O. Box 190-50100
Kakamega, Kenya

Tel: 056-31375
Fax: 056-30153
E-mail: ierc@mmust.ac.ke
Website: www.mmust.ac.ke

Institutional Ethics Review Committee (IERC)

Date: 9th September, 2019

Ref: MMU/COR: 403012 vol2 (47)

Okoth Fredrick Omondi
Masinde Muliro University of Science and Technology
P.O. Box 190-50100
KAKAMEGA

Dear Mr. Okoth

RE: Queuing analysis of hospital congestion: A case study of Moi Teaching and Referral hospital - MMUST/IERC/074 /19

Thank you for submitting your proposal entitled as above for initial review. This is to inform you that the committee conducted the initial review and approved (with minor revisions) the above Referenced application for one year.

This approval is valid from **9th September, 2019 through to 9th September, 2020**. Please note that authorization to conduct this study will automatically expire on **9th September, 2020**. If you plan to **continue with data collection or analysis beyond this date please submit an application for continuing approval to the MMUST IERC by 9th August, 2020**.

Approval for continuation of the study will be subject to submission and review of an annual report that must reach the MMUST IERC secretariat by **9th August, 2020**. You are required to submit any amendments to this protocol and any other information pertinent to human participation in this study to MMUST IERC prior to implementation.

Please note that any unanticipated problems or adverse effects/events resulting from the conduct of this study must be reported to MMUST IERC. Also note that you are required to seek for research permit from NACOSTI prior to the initiation of the study.


Yours faithfully,

Dr. Gordon Nguka (PhD)
Chairman, Institutional Ethics Review Committee


Copy to:

- The Secretary, National Bio-Ethics Committee
- Vice Chancellor
- DVC (PR&I)
- DVC (A & F)

APPENDIX II: Approval – NACOSTI



REPUBLIC OF KENYA



**NATIONAL COMMISSION FOR
SCIENCE, TECHNOLOGY & INNOVATION**

Date of Issue: **03/February/2020**

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Director General
**NATIONAL COMMISSION FOR
SCIENCE, TECHNOLOGY & INNOVATION**

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APPENDIX III: Approval – IREC MOI UNIV/MTRH



INSTITUTIONAL RESEARCH AND ETHICS COMMITTEE (IREC)

MOI TEACHING AND REFERRAL HOSPITAL
P.O. BOX 3
ELDORET
Tel: 33471/2/3

MOI UNIVERSITY
COLLEGE OF HEALTH SCIENCES
P.O. BOX 4606
ELDORET
Tel: 33471/2/3
22nd May, 2020

Reference: IREC/2020/52

Approval Number: 0003603

Mr. Fredrick O. Okoth,
Masinde Muliro University of Science & Technology,
P.O. Box 190-50100,
KAKAMEGA-KENYA.



Dear Mr. Okoth,

MODELLING QUEUING PHENOMENON OF HOSPITAL CONGESTION; A CASE STUDY OF MOI TEACHING AND REFERRAL HOSPITAL

This is to inform you that **MU/MTRH-IREC** has reviewed and approved your above research proposal. Your application approval number is **FAN: 0003603**. The approval period is **22nd May, 2020 – 21st May, 2021**.

This approval is subject to compliance with the following requirements;

- i. Only approved documents including (informed consents, study instruments, MTA) will be used.
- ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by **MU/MTRH-IREC**.
- iii. Death and life threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to **MU/MTRH-IREC** within 72 hours of notification.
- iv. Any changes, anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to **MU/MTRH-IREC** within 72 hours.
- v. Clearance for export of biological specimens must be obtained from relevant institutions.
- vi. Submission of a request for renewal of approval at least 60 days prior to expiry of the approval period. Attach a comprehensive progress report to support the renewal.
- vii. Submission of an executive summary report within 90 days upon completion of the study to **MU/MTRH-IREC**.

Prior to commencing your study; you will be required to obtain a research license from the National Commission for Science, Technology and Innovation (NACOSTI) <https://oris.nacosti.go.ke> and other relevant clearances. Further, a written approval from the CEO-MTRH is mandatory for studies to be undertaken within the jurisdiction of Moi Teaching & Referral Hospital (MTRH), which includes 22 Counties in the Western half of Kenya.

Sincerely,

DR. S. NYABERA
DEPUTY-CHAIRMAN
INSTITUTIONAL RESEARCH AND ETHICS COMMITTEE

cc CEO - MTRH Dean - SOP

Dean - SOM



INSTITUTIONAL RESEARCH AND ETHICS COMMITTEE (IREC)

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Tel: 334711/2/3
22nd May, 2020

Reference: IREC/2020/52

Approval Number: 0003603

Mr. Fredrick O.Okoth,
Masinde Muliro University of Science & Technology,
P.O. Box190-50100,
KAKAMEGA-KENYA.



Dear Mr. Okoth,

MODELLING QUEUING PHENOMENON OF HOSPITAL CONGESTION; A CASE STUDY OF MOI TEACHING AND REFERRAL HOSPITAL

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This approval is subject to compliance with the following requirements;

- i. Only approved documents including (informed consents, study instruments, MTA) will be used.
- ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by **MU/MTRH-IREC**.
- iii. Death and life threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to **MU/MTRH-IREC** within 72 hours of notification.
- iv. Any changes, anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to **MU/MTRH-IREC** within 72 hours.
- v. Clearance for export of biological specimens must be obtained from relevant institutions.
- vi. Submission of a request for renewal of approval at least 60 days prior to expiry of the approval period. Attach a comprehensive progress report to support the renewal.
- vii. Submission of an executive summary report within 90 days upon completion of the study to **MU/MTRH-IREC**.

Prior to commencing your study; you will be required to obtain a research license from the National Commission for Science, Technology and Innovation (NACOSTI) <https://oris.nacosti.go.ke> and other relevant clearances. Further, a written approval from the CEO-MTRH is mandatory for studies to be undertaken within the jurisdiction of Moi Teaching & Referral Hospital (MTRH), which includes 22 Counties in the Western half of Kenya.

Sincerely,

DR. S. NYABERA
DEPUTY-CHAIRMAN
INSTITUTIONAL RESEARCH AND ETHICS COMMITTEE

cc CEO - MTRH Dean - SOP

Dean - SOM

APPENDIX IV: Information Collection Guide

This section is designed to aid in recording key variables that will be analyzed so as to develop a queuing model that describe the present scenario as well as work out an improved model that shall ease congestion at the Emergency Department of MTRH. Accurate feedback with help in achieving this noble objective.

- (1) What is the number of critically ill patients' arrivals per day/week?
- (2) What is the number of critically ill patients are admitted per day/week?
.....
- (3) How long do they take before being admitted at the Emergency Department?
.....
- (4) How many internal wards are available?
.....
- (5) What is the patient's internal wards allocation criteria/policy?
.....
- (6) What is the number of beds per internal ward?
.....
- (7) What is the number of staff per ward (nurses, doctors, and support staff)?
.....
- (8) What is the standard and maximum occupancy per internal ward?
.....
- (9) Average length of stay per ward (days)?
.....

(10) Kindly provide Emergency Department - Internal Wards integrated (activities-resources) flow chart diagrams.

.....

(11) What are the causes of delays in the Internal Wards/ Emergency Department

.....

(12) What is the number of weekly discharge?

.....

(13) What is the rate of return for treatment by patients who have been discharged per month?

(14) What is the number of mortality per week?

.....

	Ward A	Ward B	Ward C	Ward D	Ward E	Ward Y	Ward Z
Mean waiting time to admission							
Av. Length of Stay (days)							
Mean Occupancy rate							
Mean No. of patients (month)							
Standard Capacity (Beds)							
Maximum capacity (Beds)							
Mean No. patients per bed per (month)							
Rate of return per 3 months							
Routing Policy							
Mean discharge per month							
Mean No. of Staff							
Average mortality per month							
Monthly transfers from ED - IW							

APPENDIX V: Approval – Information Collection Response

This section is designed to aid in recording key variables that will be analyzed so as to develop a queuing model that describes the present scenario as well as work out an improved model that shall ease congestion at the Emergency Department of MTRH. Accurate feedback will help in achieving this noble objective.

For questions 1 and 2 refer to table I.

1. What is the number of critically ill patients' arrivals per day/week?
2. What is the number of critically ill patients are admitted per day/week? Table

(I) below shows the Attendance and Admissions from January –March 2020

Period	Emergency Department Outpatient Attendance	Emergency Department Admissions
Jan-20	8784	1651
Feb-20	8258	1664
Mar-20	6712	1376
Total	23754	4691

4. How many internal wards were available? 28 Wards
5. What is the patient's internal wards allocation criteria/policy? The criteria for ward allocation is based on the patient's age, condition and diagnosis.
6. What is the number of beds per internal wards?
8. What is the standard and maximum occupancy per internal ward?
9. Average length of stay per ward (days)?
12. What is the number of weekly discharge?
14. What is the number of mortality per week?

For questions 4,6,8,9 & 12 refer to tables,II, III & IV which are monthly figures.

Table II showing Wards and Inpatient Statistics(Allocated beds, Admissions, Mortality and Mortality rate,Average Occupancy, Average Length of Stay and Discharges.(January 2020)

JANUARY 2020

WARDS	ALLOCATED BEDS	TOTAL ADMISSIONS	MORTALITY & MORTALITY RATE(%)	AVERAGE OCCUPANCY (%)	AVERAGE LENGTH OF STAY(ALOS) (DAYS)	DISCHARGES
AMANI	92	282	49(19%)	98	12	258
UMOJA	96	287	51(20%)	89	11	256
CCU	14	46	13(31%)	11	8	42
ISOLATION WARD	5	1	0(%)	4	0	0
FARAJA	40	251	14(6%)	53	7	244
RILEY MOTHER & BABY	125	1186	4(0%)	31	1	955
NEEMA	60	258	56(23%)	89	11	248
UPENDO	32	163	11(7%)	45	9	147
TUMAINI	35	144	12(%)	50	11	143
SUBIRA	35	130	5(4%)	51	13	123
FADHILI	27	160	1(1%)	42	9	146
BURNS PEADS	22	19	1(6%)	19	34	17
NEURO PEADS	11	29	1(3%)	15	14	32
NEURO (MALE)	20	105	14(11%)	46	11	124
NEURO (FEMALE)	10	29	3(10%)	10	11	29
LONGONOT	35	123	2(1%)	73	16	146
SERGOIT	12	53	2(4%)	18	12	47
KILIMANJARO	49	160	14(10%)	69	15	141
REHEMA	46	108	10(10%)	29	9	100
ICU	21	126	57(47%)	19	5	122
HDU	3	0	0(0%)	0	0	0
ELGON	24	80	0(0%)	13	6	74
KENYA	80	108	0(0%)	82	22	116
ADA	16	1	0(0%)	16	162	3
PW I	51	165	6(4%)	40	8	146
PW II ADULT	38	139	6(4%)	34	8	135
PW II PEADS	6	62	0(0%)	10	5	61
PW II MAT	12	69	0(0%)	7	3	74
PW II NBU	3	2	1(33%)	0	2	3
TOTAL	1020	4065	333(9%)	1063	9	3721

Table III showing Wards and Inpatient Statistics(Allocated beds, Admissions, Mortality and Mortality rate,Average Occupancy,Average Length Stay and Discharges.)

FEBRUARY 2020

WARDS	ALLOCATED BEDS	TOTAL ADMISSIONS	MORTALITY & MORTALITY RATE(%)	AVERAGE OCCUPANCY (%)	AVERAGE LENGTH OF STAY(ALOS) (DAYS)	DISCHARGES
AMANI	92	260	57(22%)	101	11	262
UMOJA	96	281	56(19)	92	9	291
CCU	14	46	8(17%)	15	9	47
ISOLATION WARD	5	0	0(0%)	4	104	1
FARAJA	40	263	9(4%)	59	7	248
RILEY MOTHER & BABY	125	1159	1(1%)	22	1	1166
NEEMA	60	273	52(20%)	104	12	260
UPENDO	32	147	8(5%)	47	9	150
TUMAINI	35	138	14(11%)	52	12	128
SUBIRA	35	133	10(7%)	55	12	136
FADHILI	27	164	1(1%)	52	9	171
BURNS PEADS	22	23	0(0%)	24	45	15
NEURO PEADS	11	27	2(9%)	11	14	23
NEURO (MALE)	20	80	7(9%)	36	14	78
NEURO (FEMALE)	10	21	3(14%)	10	13	21
LONGONOT	35	124	2(2%)	58	14	121
SERGOIT	12	49	0(0%)	21	13	48
KILIMANJARO	49	201	9(5%)	76	11	198
REHEMA	46	101	13(15%)	35	12	86
ICU	21	78	28(36%)	19	7	77
HDU	3	10	0(0%)	1	4	10
ELGON	24	55	0(0%)	11	5	59
KENYA	80	92	0(0%)	81	27	87
ADA	16	10	0(0%)	16	51	9
PW I	51	156	7(4%)	43	8	167
PW II ADULT	38	123	7(6%)	34	8	120
PW II PEADS	6	51	0(0%)	9	5	51
PW II MAT	12	75	0(0%)	8	3	74
PW II NBU	3	3	0(0%)	1	12	2
TOTAL	1020	3942	294(8%)	1098	8	3910

Table IV showing Wards and Inpatient Statistics (Allocated beds, Admissions, Mortality and Mortality rate, Average Occupancy, Average Length Stay and Discharges.)

MARCH 2020

WARDS	ALLOCATED BEDS	TOTAL ADMISSIONS	MORTALITY & MORTALITY RATE(%)	AVERAGE OCCUPANCY (%)	AVERAGE LENGTH OF STAY(ALOS)	DISCHARGES
AMANI	92	218	48(18%)	82	10	265
UMOJA	96	239	56(20%)	81	9	275
CCU	14	40	6(14%)	11	8	44
ISOLATION WARD	5	1	1(100%)	3	99	1
FARAJA	40	195	2(1%)	46	6	236
RILEY MOTHER & BABY	125	1139	4(0.345)	33	1	1148
NEEMA	60	236	38(14%)	97	11	276
UPENDO	32	120	11(8%)	44	10	139
TUMAINI	35	131	15(9%)	46	9	158
SUBIRA	35	152	4(2%)	48	9	166
FADHILI	27	145	3(2%)	34	6	163
BURNS PEADS	22	21	0(0%)	25	33	24
NEURO PEADS	11	27	2(5%)	12	9	39
NEURO (MALE)	20	80	2(2%)	31	9	103
NEURO (FEMALE)	10	26	2(7%)	10	11	30
LONGONOT	35	116	0(0%)	45	10	145
SERGOIT	12	44	1(2%)	18	11	52
KILIMANJARO	49	128	14(8%)	51	9	175
REHEMA	46	82	10(9%)	36	10	108
ICU	21	83	37(44%)	19	7	85
HDU	3	0	0(0%)	0	0	0
ELGON	24	45	0(0%)	7	5	50
KENYA	80	92	1(1%)	80	20	127
ADA	16	1	0(0%)	14	87	5
PW I	51	148	18(11%)	39	7	171
PW II ADULT	38	113	9(7%)	30	7	125
PW II PEADS	6	58	1(2%)	11	6	60
PW II MAT	12	51	0(0%)	5	3	54
PW II NBU	3	1	0(0%)	0	5	2
TOTAL	1020	3570	285(7%)	959	7	4039

- NB: PWI stands for Private Wing I & PW II stands for Private Wing II.
 - All the data captured above are the monthly figures from January 2020 to March 2020.
 - In the Mortality & Mortality rate column, the figures outside the brackets represent the mortality while the figures inside the brackets represent the mortality

rate in that particular ward.

- The Average Occupancy and Mortality rates are in percentages while the Average length of Stay is in days.

- The data provided should assist you in Questions 1,2,4,5,,6,8,9,12,13, & 14.

Prepared by 28/07/2020

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